Neutrino Mass Mechanisms and Leptogenesis

1. neutrino masses (majorana or dirac) and mixing angles

2. mechanisms for small Dirac masses

3. mechanisms for small Majorana masses
   - suppressed by a large mass scale and small couplings: the seesaw
   - suppressed by small couplings and loops: $R_p$ violation in SUSY

4. leptogenesis
   - required ingredients for baryogenesis
   - baryogenesis via leptogenesis
   - flavoured thermal leptogenesis type I seesaw, hierarchical $N_i$
The Baryon Asymmetry of the Universe

we see the earth, the solar system, the galaxy, galaxy clusters, \( \Rightarrow \) are made of matter (no \( \gamma \) rays from annihilation)

\[ \Rightarrow \text{there is an excess of matter over anti-matter in the Universe.} \]

matter \( \simeq H = p + e^- \), so this implies a baryon asymmetry:

\[ 7\frac{n_B - n_{\bar{B}}}{s} \simeq \frac{n_B - n_{\bar{B}}}{n_{\gamma}} \simeq \begin{cases} \sim \text{few} \times 10^{-11} & \text{luminous} \\ 2 - 6 \times 10^{-10} & \text{BBN} \\ 6 \times 10^{-10} & \text{WMAP} \end{cases} \]

(not worry about lepton asymmetry, because there is an undetectable C(M)B of \( \nu \)s, which could contain a large asymmetry.)

could the Universe have been \textit{born} with a baryon asymmetry?
No: asymmetry exponentially diluted during inflation (required for \( \Delta T/T \)).

\[ \Rightarrow \text{we must create the asymmetry after inflation} \]
parenthese: how to measure $Y_B = (n_B - n_B^\bar{\bar{\bar{\bar{\bar{\bar{\bar{\bar{\bar{\bar{B}}}}}}}}}})/n_\gamma$?

$n_\gamma$ photon number density, from CMB.

- luminous matter — we see it.
- BBN
  - baryons rare $\rightarrow$ make nucleons in 2-body processes,
  - density of $D_e$ ($E_{bind} \sim 2.2$ MeV) develops when not disassociated by energetic $\gamma$ from Boltzmann tail:

$$n_B - n_B^\bar{\bar{\bar{\bar{\bar{\bar{\bar{\bar{\bar{\bar{B}}}}}}}}}} \gtrsim n_\gamma(E > 2.2\text{MeV}) \sim E^3 e^{-2.2\text{MeV}/T_{BBN}} \Rightarrow -\ln Y_B \sim 2.2\text{MeV}/T_{BBN}$$

- at $T_{BBN} \sim 0.1\text{MeV}$, make $D_e$ with all available $n$, then $^4\text{He}$ with all available $D_e$
- (but $n$ are decaying to $p$, $\tau_n \sim 10$ minutes, $\tau_U \sim (T/\text{MeV})^2$ seconds)
- amount of $^4\text{He} \rightarrow$ temperature of BBN $\rightarrow n_B - n_B^\bar{\bar{\bar{\bar{\bar{\bar{\bar{\bar{\bar{\bar{B}}}}}}}}}}$
- CMB most accurate
  - before recombination, within horizon, photon-baryon fluid oscillates (sound waves). These are the peaks in the $C_\ell$ plots.
  - amplitude of oscillation $\leftrightarrow$ baryon density
  - $\Rightarrow$ height of first peak related to $Y_B$. 
Required ingredients to make a baryon asymmetry

1. B violation
   if Universe \( (\equiv U) \) starts in state of \( n_B - n_{\bar{B}} = 0 \), need \( B \) to evolve to \( n_B - n_{\bar{B}} \neq 0 \)

2. C and CP violation
   if \( U \) starts in CP eigenstate, need \( CP \) in evolution to obtain excess of particles over anti-particles

3. out-of-thermal-equilibrium dynamics
   equilibrium = static, no asymmetries in unconserved quantum numbers

   \textbf{in the Standard Model??}

2. C and CP violation — in the CKM matrix
3. out-of-thermal-equilibrium dynamics — \( U(\text{iverse}) \) is expanding and cooling, so \( T\bar{E} \) from

   - slow interactions: \( \tau_{int} \gg \tau_{U} \left( \Gamma_{int} \ll H \right) \)
     \[ \text{but... } \Gamma_{\text{decay}} \sim \frac{\lambda^2 M}{8\pi} \]
     \[ H \sim \frac{T^2}{m_{pl}} \sim 10^{-17} T\bigg|_{T=m_W} \]

   - phase transitions: the electroweak phase transition
B+L violation in the Standard Model

1. Baryon number —
   is experimentally conserved
   is conserved in the renormalisable SM Lagrangian
...BUT: your friend the axial anomaly, who gives $\pi^0 \rightarrow \gamma\gamma$, also gives $B + L$ interactions that are fast at $T > m_W$. In field theory of massless chiral fermions:

$$\partial_\mu J_5^\mu = \overline{\psi}_R \gamma^\mu \partial_\mu \psi_R - \overline{\psi}_L \gamma^\mu \partial_\mu \psi_L = \left\{ \begin{array}{ll} 0 & \text{classical theory (no loops)} \\ \propto F \tilde{F} & \text{“winding number”, when renormalise} \end{array} \right.$$
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For LH SU(2) doublet $\psi^i_L$ of the SM:

$$\partial^\mu (\overline{\psi}^i_L \gamma_\mu \psi^i_L) = \frac{1}{64\pi^2} W^A_{\mu\nu} \tilde{W}^{\mu\nu A}.$$ 

If define $Q^i(t) = \int \overline{\psi}^i_L \gamma_0 \psi^i_L d^3x$, $\Delta Q^i = Q^i(+\infty) - Q^i(-\infty)$:

$$\Delta Q^i = \frac{1}{64\pi^2} \int d^4x W^A_{\mu\nu} \tilde{W}^{\mu\nu A}$$

then gauge field configuration of non-zero winding number acts as source of fermions.
Baryon number — is experimentally conserved
is conserved in the renormalisable SM Lagrangian

But: there are SU(2) gauge field configurations of non-zero winding number that produce one of every fermion doublet. How fast are they?
SM B+L violation: rates

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A tunneling process ("instanton") at $T = 0$, $\Gamma \propto e^{-8\pi/g^2}$. (?).

At $0 < T < T_{EPT}$, can climb over the barrier… (? maybe no barrier above EPT?)

\[ \Gamma_{B+L} \sim \alpha^5 T e^{-mW/T} \quad T > EWPT \]

\[ T < EWPT \]

IN equilibrium after the SM electroweak phase transition

no EW baryogenesis in the standard model
but fast SM $B$ at $T > EPT$
summary, caveats

• one measured number: the baryon to photon ratio

\[ Y_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6 \pm 1) \times 10^{-10} \ (CMB) \]

• required ingredients \((B, CP, TE)\) are present in the SM (of part phys and cosmo)...but *don’t* make a baryon asymmetry.

• ⇒ we need Beyond the Standard Model physics ! ...but... recreational phenomenology? any extension of the SM has many free parameters, could always tune to get \(Y_B\)?

• *pas aussi facile que ça:* other constraints on \(B, CP, TE\)
  eg proton lifetime: \(\tau_p \gtrsim 10^{32}\) yrs
  timescale for baryogenesis: \(\tau_U \approx 10^{-10} \ sec \ (EWPT) \quad 1 \ sec \ (nucleosynthesis)\)

⇒ we would like : BSM physics that is otherwise motivated (data, theoretically attractive), and generates the baryon asymmetry without making the proton decay (*e.g.* “sphalerons”).

⇒ *THE (SUSY) SEESAW*
The See-Saw in three generations

• in the charged lepton ("flavour") and $N(=\nu_R)$ mass bases, at large energy scale $\gg M_i$:

$$\mathcal{L} = \mathcal{L}_{SM} + \lambda_{\alpha J}^* \bar{\ell}_\alpha \cdot H N_J - \frac{1}{2} N_J M_J N_J^c$$

18 - 3 ($\ell$ phases) in $\lambda$

$$m_\alpha = m_e, m_\mu, m_\tau, M_1, M_2, M_3$$

21 parameters chez les leptons:

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• at the weak scale, get effective light neutrino mass matrix

$$\lambda M^{-1} \lambda^T \langle H^0 \rangle^2 = [m_\nu] = U^* D_m U^\dagger$$

12 parameters:

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$$m_e, m_\mu, m_\tau, m_1, m_2, m_3$$

6 in $U_{MNS}$
The Matter Excess of the Universe—Leptogenesis

C and CP violation (complex $\lambda$)
Recall: required ingredients: non-equilibrium dynamics ($\Gamma_N < H$)
baryon number violation ($\mathcal{L}_B + \mathcal{L}_L$)

three steps:

1. **dynamics** produce some number density of $N$, who later ($T \lesssim M_1$) decay
2. If the $N$ interactions are **CP** violating, a lepton asymmetry $Y_L$ may be produced
3. non-perturbative $\mathcal{B} + \mathcal{L}$ SM processes: lepton asymmetry $\rightarrow$ baryon asymmetry.
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many implementations:

- thermal leptogenesis with hierarchical $M_J$: $M_1 \ll M_{2,3}$
- thermal leptogenesis with quasi-degenerate $M_J$ ($C\overline{P}$ in mixing)
- “soft leptogenesis” ($\tilde{N}$ decay, soft SUSY terms give $C\overline{P}$ in mixing $\tilde{N}_J - \tilde{N}_J^*$)
- a la Affleck Dine (classical field evolution in the early $U$)
- non-thermal $N$ production (from inflaton decay, in preheating, ...)
- ...also Dirac leptogenesis...
- ...

... falsifiable ???
Another tangent: why hierarchical $N_j$?

In the (type 1) seesaw:

$$[m_\nu] = [\lambda]^T [M]^{-1} [\lambda] v_u^2$$

take determinants:

$$m_3 m_2 m_1 = \frac{\lambda_1^2 \lambda_2^2 \lambda_3^2}{M_1 M_2 M_3} v_u^6$$

• assume a steep hierarchy in the Yukawas $\lambda_3 \sim 1$, $\lambda_2 \sim 10^{-2}$, $\lambda_1 \sim 10^{-4}$

$$m_3^2 m_2 m_1 = \frac{10^{-12}}{M_1 M_2 M_3^3} v_u^6$$

• assume that (!) $m_3 \sim v_u^2 / M_3$ is natural

$$\frac{m_2 m_1}{m_3 m_3} = \frac{10^{-12}}{M_1 M_2 M_3^3}$$

• assume $m_1 \gtrsim .1 \div .01 m_2$ (not $\sim 10^{-10} m_2$)

$$\frac{.1 m_1}{m_3} = \frac{10^{-12}}{M_1 M_2 M_3^3}$$

...so to get mild hierarchy in $m_i$, given steep hierarchy in $\lambda_i$ and degenerate $M_J$...requires a “conspiracy” in RH sector (eg $\pi/4$ mixing angles)
After inflation, vacuum energy density is transferred to a hot thermal soup at $T_{\text{reheat}}$, containing particles with gauge interactions (no $N_i$). Then...

1. somehow produce some number density $Y_{N_1}$ of $N_1$ (helpful to have $M_1 \lesssim T_{\text{reheat}}$).
   Later (once $Y_{N_1} \simeq Y_{N_1}^{eq}$), the $N_1$ will decay away.

2. If the $N_1$ decay is $CP$: $\Gamma(N_1 \rightarrow H\ell_\alpha) - \Gamma(N_1 \rightarrow \bar{H}\ell_\alpha) \neq 0$, an asymmetry in $L_\alpha$ can be produced
   If the inverse decays are “out of equilibrium”, the asymmetry could survive.

3. SM non-perturbative $\mathcal{B} + \mathcal{L}$ partially transforms lepton asym $\rightarrow$ baryon asym.
Thermal leptogenesis with hierarchical $N_i$

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2. If the $N_1$ decay is $CP$-violating: $\Gamma(N_1 \to H\ell_\alpha) - \Gamma(N_1 \to \bar{H}\ell_\alpha) \neq 0$, an asymmetry in $L_\alpha$ can be produced $\Leftrightarrow \epsilon_{\alpha\bar{\alpha}}$
   If inverse decays are “out of equilibrium”, the asymmetry could survive. $\Leftrightarrow \eta_\alpha$

3. SM non-perturbative $\bar{R} + \bar{L}$ partially transforms lepton asym $\rightarrow$ baryon asym. $\Leftrightarrow C$

Usual parametrisation ($s =$ entropy density))

$$\left( \frac{n_B - n_{\bar{B}}}{s} = \right) Y_{\Delta B} = \frac{n_{N_1}^{eq}(T \gg M_1)}{s} \sum_\alpha \frac{n_{\ell_\alpha} - n_{\bar{\ell}_\alpha}}{n_N} \times \eta_\alpha \times C.$$  

$$\sim 4 \times 10^{-3} \sum_\alpha \epsilon_{\alpha\bar{\alpha}} \times \eta_\alpha \times \frac{1}{3}$$  

$$\sim 8 \times 10^{-11}$$

$\eta$ parametrises difficulty of obtaining initial thermal number density, and out-of-equilibrium decay....
Step 2: $\mathcal{CP}$ and $\epsilon$

After inflation, vacumm energy density is transfered to a hot thermal soup at $T_{reheat}$. Then...

1. Suppose a distribution of $N_1$ (and $\bar{N}_1$) is produced, then decays away (as $T \lesssim M_1$). Departure from equilibrium required — more later.

2. If there is $\mathcal{CP}$ in decays ($\mathcal{L}$ from $M$), can produce asymmetries in lepton flavours:

$$\frac{n_{\ell\alpha} - n_{\bar{\ell}\alpha}}{n_N} \simeq \epsilon_{\alpha\alpha} = \frac{\Gamma(N_1 \to H\ell\alpha) - \Gamma(N_1 \to \bar{H}\bar{\ell}\alpha)}{\Gamma(N_1 \to H\ell) + \Gamma(N_1 \to \bar{H}\bar{\ell})}$$
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$$\frac{n_\ell - n_{\bar{\ell}}}{n_N} \approx \epsilon_{\alpha\alpha} = \frac{\Gamma(N_1 \to H\ell_\alpha) - \Gamma(N_1 \to \bar{H}\bar{\ell}_\alpha)}{\Gamma(N_1 \to H\ell) + \Gamma(N_1 \to \bar{H}\bar{\ell})}$$

To obtain $\Gamma - \bar{\Gamma} \neq 0$, need $\text{Im} \{ \text{coupling constants } c \} \times \text{Im} \{ \text{“amplitude” } A \}$.

Write tree + loop: $\mathcal{M} = c_0 A_0 + c_1 A_1$, $\overline{\mathcal{M}} = c_0^* A_0 + c_1^* A_1$ (sloppy)

$$\Rightarrow \int d(\text{phase space}) \left| (\mathcal{M})^2 - |\overline{\mathcal{M}}|^2 \right| \propto \text{Im} \{ c_0 c_1^* \} \text{Im} \{ A_0 A_1^* \}$$

$\text{Im} \{ A \} \Leftrightarrow$ on-shell intermediate particles in a loop:
Guestimating $\epsilon_{\alpha\alpha}$

$\ell$ in $N_1$ decays from $\{\lambda, M\}$, and $CP$, allows to produce asymmetry $\epsilon_{\alpha\alpha} \simeq (n_{\ell\alpha} - n_{\bar{\ell}\alpha})/n_N$:

\[
\epsilon_{\alpha\alpha} = \frac{\Gamma(N_1 \to H\ell_\alpha) - \Gamma(N_1 \to \bar{H}\bar{\ell}_\alpha)}{\Gamma(N_1 \to H\ell_\alpha) + \Gamma(N_1 \to \bar{H}\bar{\ell}_\alpha)} \simeq \frac{-3M_1}{8\pi [\lambda^\dagger \lambda]_{11}} \text{Im} \left[ \lambda^T \left[ \frac{m_\nu}{v_u^2} \right]_{1\alpha} \lambda_{\alpha1} \right] < \frac{3}{8\pi} \frac{M_1 m_{\nu,\text{max}}}{v_u^2}
\]

lower bound on $M_1 \gtrsim T_{\text{reheat}}/5$ to get a big enough asymmetry:

$M_1 \gtrsim 10^9\text{GeV}$  \hspace{1cm} \text{gravitinos!??}
Recall the scenario:

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- $N_1$ is gauge singlet, produced and decays via Yukawa $\lambda$.
  
  $\Gamma_{prod} \sim h_t^2\lambda^2T > H(T \sim M_1)$ to reach $n_N \sim n_\gamma$
  
  $\Gamma_{dec} \sim \lambda^2T < H(T \lesssim M_1)$ to have out of equilibrium decay

  $\Rightarrow$ **DOES IT WORK??**  
  
  **Yes!** $\Rightarrow$ BoltzmannEqns

_Buchmuller Plumacher_
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  Yes! $\Rightarrow$ **BoltzmannEqns**

- “washout” interactions (**e.g.** $H\ell_\alpha \rightarrow N_1$, they eat the asym), are required for thermal leptogenesis. Their strength is different for different lepton flavours

  $\Rightarrow$ **FLAVOURED LEPTON ASYMS**
Non equilibrium ($\eta$) and why a Boltzmann code

1. produce the (maximal) thermal density $n_N \simeq n_\gamma$ if $(M_1 \lesssim T$, and) production rate, e.g. $\Gamma(q_L t_R^c \rightarrow \phi \rightarrow \ell N)$ is fast enough ($\tau_{prod} < \tau_U$):

$$\Gamma_{prod} \sim \frac{h^2_t [\lambda \lambda^\dagger]_{11}}{4\pi} T > H, \quad \Rightarrow \quad [\lambda \lambda^\dagger]_{11} > \frac{10T}{m_{pl}} \bigg|_{T=M_1}$$

Can show: $\frac{[\lambda \lambda^\dagger]_{11} v_u^2}{M_1} = \tilde{m}_1 > m_1$. "Expect" $\tilde{m}_1 \gtrsim m_{sol}, \Rightarrow \Gamma_{prod} \simeq \Gamma_{decay} \gg H$

need Boltzmann code... but here make analytic estimates...
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2. The lepton asym in flavour $\alpha$ (produced from $N$ decay) can survive after Inverse Decays from flavour $\alpha$ turn off $(\tau_{ID} > \tau_U)$

$$\Gamma_{ID,\alpha} \equiv \Gamma(\ell_\alpha \phi \rightarrow N_1) \sim \Gamma_{decay}(N_1 \rightarrow \ell_\alpha H) e^{-M_1/T} \sim \frac{|\lambda_\alpha|_1^2 M_1 e^{-M_1/T}}{8\pi} \lesssim \frac{10T^2}{m_{pl}}$$
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Then, at temperature $\equiv T_\alpha$ when Inverse Decays from flavour $\alpha$ turn off,

$$\frac{n_N}{n_\gamma}(T_\alpha) \simeq e^{-M_1/T_\alpha} \simeq \frac{H}{\Gamma(N_1 \to \ell_\alpha \phi)} \equiv \eta_\alpha$$
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   \]
   Then, at temperature $\equiv T_\alpha$ when Inverse Decays from flavour $\alpha$ turn off,
   \[
   \frac{n_N(T_\alpha)}{n_\gamma} \simeq e^{-M_1/T_\alpha} \simeq \frac{H}{\Gamma(N_1 \rightarrow \ell_\alpha \phi)} \equiv \eta_\alpha
   \]
   so, if have CP asym $\epsilon_{\alpha\alpha} \neq 0, \epsilon_{\beta\beta} = 0$:
   \[
   Y_B \sim \frac{1}{3} \frac{n_{\ell_\alpha} - n_{\bar{\ell}_\alpha}}{n_N} \frac{n_N(T_\alpha)}{n_\gamma} \sim \frac{1}{3} \epsilon_{\alpha\alpha} \frac{H}{g^* \Gamma_{\text{decay}}(N_1 \rightarrow \ell_\alpha \phi)}
   \]
   (sum over flavour in general case)
Non-equilibrium: why/when Flavour Asymmetries $L_C$

- a population of (the lightest) $\nu_R$s is produced via its Yukawa coupling (e.g. $q t^c \rightarrow \nu_R \ell_\alpha$, $\phi \ell_\alpha \rightarrow \nu_R$).

- Population later disappears via same Yukawa coupling (e.g. $\nu_R \rightarrow \phi \ell_\alpha$...)

- there is CP violation in production and disappearance...

  $\Rightarrow$ asymmetry in lepton number made with the $\nu_R$ is exactly opposite to asymmetry made when $\nu_R$ go away (In the case I calculated)

  $\Rightarrow$ thermal leptogenesis “works”, because there are Yukawa interactions of the $\nu_R$ (e.g. inverse decays $\phi \ell_\alpha \rightarrow \nu_R$ between production and disappearance of $\nu_R$ population, call these interactions washout. They deplete the lepton asymmetry made with the $\nu_R$s.

  For instance, when $\nu_R$ interactions are fast, washout is effective, and the asym made with $\nu_R$s is completely destroyed.

- flavour matters, because washout does: initial state for washout interactions contains a SM LH lepton, so must know are leptons distinguishable?

  compare rates for charged lepton Yukawas $h_\tau$, $h_\mu$ to $H$, $\Gamma(\nu_R \rightarrow \ell \phi)$.

  If “in equilibrium”, Yukawas contribute to “thermal masses” $\Leftrightarrow$ distinguish flavours

$$\Gamma_\tau \simeq 10^{-2} h_\tau^2 T > H \text{ for } T < 10^{12} \text{ GeV, } \Gamma_\mu > H \text{ for } T < 10^9 \text{ GeV}$$

In washout rates:

- distinguishable $\Rightarrow$ sum probabilities
- indistinguishable $\Rightarrow$ sum amplitudes
**What changes phenomenologically, including flavour?**

“single flavour” approx, successful thermal leptogenesis $\Rightarrow$ light $\nu$ mass scale $\lesssim .1$ eV.

“flavoured”: more $C\mathcal{P}$, so no bound. Models can be tuned to work for $m_\nu \lesssim$ few eV (cosmo)

There is an envelope, in space of parameters leptogenesis depends on $(M_1, \Gamma, \epsilon...)$ where leptogenesis can work.

Including flavour gives envelope more dimensions $(M_1, \epsilon_{\alpha\alpha}, \Gamma_{\alpha\alpha})$, little changes to “interesting” regions of the envelope projected onto $M_1$, $\Gamma$ space (not move lower bound on $T_{\text{reheat}}$)

“single flavour”: no model-indep connection between $C\mathcal{P}$ for leptogenesis and MNS phases.

“flavoured”: still no sensitivity of baryon asym. to MNS phases (but can say things in classes of models)
But use flavoured estimates to check if your model works...

The baryon to entropy ratio, as a function of “time”, in flavoured and unflavoured calculation.

\[ \epsilon_{\tau\tau} = 2.5 \times 10^{-6}, \quad \epsilon_{\mu\mu} = -2 \times 10^{-6}, \quad \epsilon_{ee} = 10^{-7} \]

\[ M_1 = 10^{10} \text{ GeV} \]

\[ \frac{\Gamma_{\tau\tau}}{H} \approx 10, \quad \frac{\Gamma_{\mu\mu}}{H} \approx 30, \quad \frac{\Gamma_{ee}}{H} \approx 30 \]
Observations that would *support* thermal leptogenesis

Suppose that we *DO* observe

1. $m_\nu$ is majorana from $0\nu2\beta$ expts
   - this is a prediction of the seesaw...

2. $\mathcal{CP}$ in neutrino oscillations
   - need $\mathcal{CP}$ in leptons for leptogen

3. SUSY at the LHC, and lepton flavour violation (LFV), like $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$ ...
   - LFV at observable rates is an expectation in the SUSY seesaw. In MSUGRA, these rates give additional information about seesaw parameters

4. ???can $T_{\text{reheat}}$ be measured?
   - If $T_{\text{reheat}} > 10^9$ GeV, consistent with the thermal leptogenesis with $M_1 \ll M_2 \ll M_3$
But what if...

1. $m_\nu$ is Dirac
   - hmm. Minimal Type 1 seesaw scenario is dead.
     But there is Dirac leptogenesis. Or, consider more (6?) singlets?

2. no CP in neutrino oscillations
   - but there are 6 phases in the seesaw, always possible to arrange unmeasurable phases to get big enough asym.

3. SUSY at the LHC, but no lepton flavour violation (LFV), such as $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$ ...
   - can fit the SUSY seesaw and working leptogenesis, to all LFV observations

4. ???can $T_{\text{reheat}}$ be measured?
   - If $T_{\text{reheat}} \ll 10^9$ GeV, thermal leptogenesis with $M_1 \ll M_2 \ll M_3$ scenario is dead.
     But...leptogenesis with degenerate $M_i$ works at an temperature...

Careful about model scans in particle physics papers: endemic prior dependence...
Summary: a fairy tale for physicists

Once upon a time, a Universe was born. (Maybe ours?)

At the christening of the Universe, the Standard Model and the Seesaw (heavy sterile $N_j$ with $\nu$ masses and $CP$ interactions) were among the gifts given by the good fairies to the Universe.

The adventure begins after inflationary expansion of the Universe:

1. Assuming its hot enough, a population of $N_1$ appear, because they like the heat.

2. As the temperature drops below $M_1$, the $N_1$ population decays away.
3. In the $CP$ and $\nu$ interactions of the $N$, an asymmetry in SM leptons is created.

4. If this asymmetry can escape the big bad wolf of thermal equilibrium...
5. the lepton asym gets partially reprocessed to a baryon asym by non-perturbative $B + L$ -violating SM processes ("sphalerons")

And the Universe lived happily ever after, containing many photons. And for every $10^{10}$ photons, there was an excess of 6 baryons (protons or neutrons), with respect to anti-baryons.