Medium modifications of hadrons in dense nuclear and hadronic matter
0- General introduction: in-Medium Hadrons

Nuclear Physics
Hadron structure

IN-MEDIUM HADRONS

Dense and hot matter

Experimentally

Intermediate energy machines (1 GeV)

Relativistic heavy ion collisions

Theoretically

Chiral dynamics
Many-body problem

In-medium hadrons

- Chiral restoration
- Nucleon structure/confinement
- Lattice QCD
- Renormalization group
**Chiral symmetry breaking**

*Pions (kaons): Goldstone bosons*

*Quark condensate: order parameter (magnetisation)*

**Equation of state at finite T and ρ**

\[
\Omega(V, T, \mu_B) = -T \ln Z(V, T, \mu_B)
\]

**Chiral symmetry restoration**

\[
\langle \bar{q}q \rangle(\mu_B, T) = \langle \bar{q}q \rangle_{\text{vac}} + \sum_n \rho \rho_n(\mu_B, T) \frac{\sigma_h}{2m_q}
\]

**Modifications of QCD vacuum**

*Hadrons = elementary excitations also modified*

**HADRONIC SPECTRAL FUNCTIONS**
**Hadron spectral functions and current-current correlators**

Hadrons spectral functions defined from the correlation function between two currents or fields having the quantum numbers of the hadron and taken at two different space-time points.

**Retarded and chronological correlators or propagators**

\[
\Pi_R(q) = -i \int d^4x e^{iq \cdot x} \Theta(x_0) \langle \langle J(x), J(0) \rangle \rangle \quad \Pi(q) = -i \int d^4x e^{iq \cdot x} \langle \langle T(J(x), J(0)) \rangle \rangle
\]

**Spectral functions**

\[
\frac{W(q)}{2\pi} = \frac{\left(-\frac{1}{\pi}\right) Im \Pi_R(q)}{e^{\beta q_0} - 1} = \frac{\left(-\frac{1}{\pi}\right) Im \Pi(q)}{e^{\beta q_0} + 1}
\]

\[
W(q) = \sum_i \frac{e^{-\beta E_i}}{Z} |\langle f | J(0) | i \rangle|^2 (2\pi)^4 \delta(q_0 + p_f - p_i) \delta^{(3)}(q + p_f - p_i)
\]

- Response to a probe which couples to a current \(J(x)\) carrying the quantum numbers of a hadron.
- Accessible experimentally at energy momentum transfer \((\omega, q)\)
Hadron spectral functions: from vacuum to medium

In the medium: Spectral functions of chiral partners should converge: Chiral dynamics?

<table>
<thead>
<tr>
<th>Fluctuation currents</th>
<th>Hadrons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle J_s(x)J_s(0) \rangle - \langle J_{ps}(x)J_{ps}(0) \rangle$</td>
<td>$M_6(600) \rightarrow M_\pi(138)$</td>
</tr>
<tr>
<td>$\langle V^{a}<em>{\mu}(x)V^{a}</em>{\mu}(0) \rangle - \langle A^a_{\mu}(x)A^a_{\mu}(0) \rangle$</td>
<td>$M_{s1}(1250) \rightarrow M_{\rho}(770)$</td>
</tr>
<tr>
<td>$\langle \Psi_+(x)\Psi_+(0) \rangle - \langle \Psi_-(x)\Psi_-(0) \rangle$</td>
<td>$M_{s11}(1535) \rightarrow M_{\rho}(938)$</td>
</tr>
</tbody>
</table>

**Figure:**
- **RHO SPECTRAL FUNCTION:** $|\mu_B| = 408$ MeV
- **Fluctuation currents** and **Hadrons**
- **THERMAL SUSCEPTIBILITY**
- **SCALAR** and **PSEUDOSCALAR**

$q = 0.3$ GeV
Outline

I- The role of chiral symmetry
   1- Basics on chiral symmetry
   2- Chiral effective theories
   3- Chiral restoration

II- Some examples of medium effects
   1- Pions, sigmas and rho mesons in nuclear and hadronic matter
   2- Omega mesons in nuclear matter
   3- Kaons in dense matter

III- In-medium hadrons and the equation of state
   1- Motivation
   2- In-medium chiral perturbation theory
   3- The role of hadron substructure and confinement
   4- Chiral effective theory and confinement
   5- Towards high densities and/or temperatures

IV- Low mass dileptons production in relativistic heavy ion collisions
   1- Theoretical approaches
   2- Comparison with data
I-The role of Chiral symmetry
1- Basics on chiral symmetry

a- Vector and axial symmetry: chiral symmetry

QCD lagrangian (without glue):

$$\mathcal{L}_{QCD} = i\overline{\psi}_u \gamma^\mu \partial_\mu \psi_u + i\overline{\psi}_d \gamma^\mu \partial_\mu \psi_d - m_u \overline{\psi}_u \psi_u - m_d \overline{\psi}_d \psi_d$$

$$= i\overline{\psi} \gamma^\mu \partial_\mu \psi - \frac{m_u + m_d}{2} \overline{\psi} \psi - \frac{m_u - m_d}{2} \overline{\psi} \tau_3 \psi$$

$$\mathcal{L}_{QCD} = i\overline{\psi}_L \gamma^\mu \partial_\mu \psi_L + i\overline{\psi}_R \gamma^\mu \partial_\mu \psi_R - m (\overline{\psi}_L \psi_R + \overline{\psi}_R \psi_L)$$

QCD Lagrangian almost exactly invariant (m~7 MeV) under transformations in the light quark sector (u-d) acting separately on left and right quarks

$$SU(2)_L : \psi_L \rightarrow e^{i\alpha_k \frac{\tau_k}{2}} \psi_L, \quad \psi_R \rightarrow \psi_R$$

$$SU(2)_R : \psi_R \rightarrow e^{i\beta_k \frac{\tau_k}{2}} \psi_R, \quad \psi_L \rightarrow \psi_L$$

$$Q_L^k = \int \mathrm{d}x \psi_L^\dagger \frac{\tau_k}{2} \psi_L = \frac{1}{2} (Q_k - Q_k^5)$$

$$Q_R^k = \int \mathrm{d}x \psi_R^\dagger \frac{\tau_k}{2} \psi_R = \frac{1}{2} (Q_k + Q_k^5)$$

Vector (isospin) and axial charge: kind of « doubling » of isospin symmetry

$$V_k^\mu = \overline{\psi} \gamma^\mu \frac{\tau_k}{2}, \quad Q_k = \int \mathrm{d}x \psi^\dagger \frac{\tau_k}{2} \psi \equiv I_k$$

$$A_k^\mu = \overline{\psi} \gamma^\mu \gamma_5 \frac{\tau_k}{2}, \quad Q_k^5 = \int \mathrm{d}x \psi^\dagger \gamma_5 \frac{\tau_k}{2} \psi$$

BUT…….
b- Spontaneous breaking of chiral symmetry

m=0 \quad [H, Q_k]=0 \quad \text{and} \quad Q_k |0> = 0 \quad \rightarrow \quad \text{Isospin multiplets}

[H, Q^5_k]=0 \quad \rightarrow \quad \text{each hadron possesses a chiral partner ???}

**OBVIOUSLY NO !!!**

The reason is that the vacuum does not have the symmetry \( Q^5_k |0> \neq 0 \). This is the spontaneous breaking of chiral (axial) symmetry

\[ [Q_i^5, [Q_j^5, H]] = \delta_{ij} \int dx \, m \, \bar{\psi}(x) \psi(x) \]

\[ \sum_n 2E_n | < n | Q_i^5 |0> |^2 = - \int dx \, 2m \, < \bar{q}q > \]

**Goldstone theorem**

Since the axial charge commutes with \( H \), its action on the vacuum should give states having the same energy. This implies the existence of soft (i.e. massless) modes: THE PION (Goldstone theorem)

**Order Parameters**

- The pion decay constant
  \[ < 0 | A_k^\mu(x) | \pi_j(p) > = -i \delta_{j,k} f_\pi p^\mu e^{-ipx} \]

- The quark condensate
  \[ < \bar{q}q > = \frac{1}{2} < 0 | \bar{\psi}_u \psi_u + \bar{\psi}_d \psi_d |0> = \frac{1}{2} < 0 | \bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L |0> \]

**GOR relation:**

\[ m^2 \pi f^2 = -2m \, < \bar{q}q > \]

\[ f_\pi = 94\text{MeV} \]

\[ <qq> = -(220\text{MeV})^3 \]
Correlation functions associated with « chiral partners »

- Vector correlator

$$\Pi_{V}^{\mu \nu}(q) = -i \int d^4 x e^{i q \cdot x} \langle \langle T(V_{\mu}(x), V_{\nu}(0)) \rangle \rangle$$

- Axial-vector correlator

$$\Pi_{A}^{\mu \nu}(q) = -i \int d^4 x e^{i q \cdot x} \langle \langle T(A_{\mu}(x), A_{\nu}(0)) \rangle \rangle$$

If chiral symmetry were in its Wigner realization they should be identical

The associated spectral functions are accessible experimentally

$$-\frac{1}{\pi} \text{Im} \Pi_{V}^{\mu \nu}(q; T = 0) = -(q^{2} g_{\mu \nu} - q_{\mu} q_{\nu}) \rho_{V}(q^{2})$$

$$-\frac{1}{\pi} \text{Im} \Pi_{A}^{\mu \nu}(q; T = 0) = q_{\mu} q_{\nu} f_{\pi}^{2} \delta(q^{2} - m_{\pi}^{2}) - \left( q^{2} g_{\mu \nu} - q_{\mu} q_{\nu} \right) \rho_{A}(q^{2})$$

Chiral symmetry breaking is a low energy phenomena
c-An illustration: the Nambu-Jona-Lasinio model

• Basis of the model
  Quartic chiral invariant interaction ($P<\Lambda \sim 1 \text{ GeV}$) simulating
  non pertubative QCD: (3 parameters $G_1$, $\Lambda$, $m$)

  $$\mathcal{L}_{NJL} = i\bar{\psi} \gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi + \frac{G_1}{2} \left[(\bar{\psi}\psi)^2 + (i\bar{\psi} \gamma^5 \tau \psi)^2\right]$$

• Spontaneous breaking of chiral symmetry: constituents quarks

  Spontaneous symmetry breaking; the quarks acquire a mass $M \sim 350 \text{ MeV}$

  $$M = m - 2G_1 \langle\langle \bar{q}q \rangle\rangle = m + 4N_c G_1 \int_{p<\Lambda} \frac{dp}{(2\pi)^3} \frac{M}{E_p}$$

  The quark condensate is made of interacting quarks-antiquarks pairs;
  the (BCS type) ground state wave function is:

  $$|\phi(M)\rangle = C \exp \left(-\sum_{s,p<\Lambda} \gamma_{ps} b_{ps}^\dagger a_{-p-s}^\dagger \right) |\phi_0\rangle = \prod_{s,p<\Lambda} \left(\alpha_p + s\beta_p b_{ps}^\dagger a_{-p-s}^\dagger \right) |\phi_0\rangle$$
Mesons: Goldstone theorem

Mesons generated as collective $\bar{q}q$ excitations, i.e., the unitarized interaction is mediated by the corresponding meson exchange

$$q \quad T \quad q = \quad \pi, \sigma \quad \approx \frac{g_B}{q^2 - m_B^2}$$

Pion decay constant

$$A^{\mu=0} = f_\pi \frac{q^\mu}{q^2 - m_\pi^2} \cdot 1$$

Adjust $f_\pi$, $m_\pi$, $<\bar{q}q>$:

$$M = g f_\pi, \quad m_\pi^2 = m g^2 / M G_1, \quad m_\sigma^2 = 4 M^2 + m_\pi^2$$

Effective potential: integrate out quarks in the Dirac sea

$$Z = \int D\Sigma D\bar{T} \exp \left( i \int d^4x L_{NJL} \right) \left[ D\Sigma D\bar{T} \delta (\Sigma - \bar{\psi} \gamma^a) \delta (\bar{T} - i \bar{\psi} Y_5 \gamma^a \psi) \right]$$

$$Z = \int D\Sigma D\bar{T} \exp \left( i \int d^4x L_{\text{eff}} (\bar{T}, \Sigma) \right)$$

$$L_{\text{eff}} = \frac{Z(\phi)}{2} \left( \partial^\mu \Sigma \partial_\mu \Sigma + \partial^\mu \bar{T} \partial_\mu \bar{T} \right) - U(\phi)$$

$$\phi = m + \Sigma + i \bar{T} Y_5$$

$$U(\phi) = \frac{\Sigma^2 + \bar{T}^2}{4 G_1}$$

$$-2 N_c N_f \int_0^\Lambda \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + \phi^2}$$
2- Chiral effective theories

a- Fluctuations of the chiral condensate

Formulate low energy QCD directly in terms of relevant hadron degrees of freedom, i.e., the modes associated with the fluctuations of the condensate around the minimum of the effective potential.

Order parameter:

\[ \mathcal{M}^{ij} = \bar{q}_{Rj} q_{Li} \]

\[ \langle \mathcal{M}^{ij} \rangle = \Sigma \delta_{ij} \neq 0 \]

Parametrized as:

\[ \mathcal{M} = \left( \frac{\bar{\psi} \psi}{2} \right) + i \bar{\tau} \cdot \left( i \bar{\psi} \gamma_5 \frac{\tau}{2} \psi \right) = \sigma + i \bar{\tau} \cdot \pi \]

In the EFT, \( \sigma \) and \( \pi \) are the effective degrees of freedom.

Introduce « polar coordinates » \( S \) and \( \phi \):

\[ \sigma + i \bar{\tau} \cdot \pi = S U = \left( f_\pi + s \right) e^{i \bar{\tau} \cdot \phi \pi / f_\pi} \]

Pion (\( \Xi \) orthoradial mode) is the phase fluctuation of the condensate: soft Goldstone moving on the « chiral circle »

Chiral invariant scalar \( S \) field (\( \Xi \) radial mode, \( \langle S \rangle = f_\pi \)) is the amplitude fluctuation of the condensate: associated with the \( \sigma \) meson
The $S$ scalar field is associated with the sigma meson $f_0 (600)$ which is very large due to its strong decay into two pions.

It has a derivative coupling to pions: it just decouples from the low energy pion dynamics: it is integrated out (frozen) in chiral perturbation theory.

**BUT**  
(G. C. M. Ericson, P. Guichon)

It cannot be ignored as soon as chiral symmetry starts to be restored since it represents that part of the condensate which governs the evolution of the masses.

We identify it with the sigma meson of nuclear physics, i.e., the background attractive scalar field at the origin of the binding.

The hadrons get polarized in this nuclear scalar field: this provides a mechanism for nuclear saturation.

Near the (tri)-critical point the sigma mass should go to zero. Vector meson masses drop through their coupling to the condensate, i.e., the $S$ field

\[
\Delta V_{\rho,\omega} = \frac{2}{3} \Delta V_N = -\frac{2}{3} n_s \left( \frac{g_s^*}{m_{\rho}^2} - \frac{g_s^2}{m_{\omega}^2} \right)
\]

(E. V. Shuryak)
b- Chiral perturbation theory

This is an « exact » copy of QCD in the low energy sector for light particles whereas heavy particles are frozen or taken as static sources. Possible since there is a clear separation (mass gap) \( \Lambda = 4\pi f_\pi \) between light particles (Goldstone bosons) and heavy particle (\( \rho, \sigma, \omega, \ldots \)).

The radial mode \( S \) is « frozen »

The QCD lagrangian is replaced by an effective one involving the \( U \) matrix representing the pions

\[
\mathcal{L}_{QCD} \rightarrow \mathcal{L}_{\text{eff}}(U, \partial U, \partial^2 U, \ldots)
\]

Expansion of the lagrangian in powers of derivatives \( (p_\pi/\Lambda) \) and in power of the quark mass or the pion mass \( (m_\pi/\Lambda) \)
• Leading term

\[ \mathcal{L}^{(2)} = \frac{f^2}{4} \text{Tr}[\partial_{\mu} U^{\dagger} \partial^\mu U] + \frac{f^2}{2} B_0 \text{Tr}[m(U + U^{\dagger})] \]

The first term is highly constrained by symmetry (QCD LσM,NJL): f = f_\pi.

The « mass » term is not universal and depends on the (chiral symmetry breaking) QCD dynamics.

\[ \langle \bar{u}u \rangle = \langle \bar{d}d \rangle = -f^2 B_0 \text{ in the chiral limit} \quad m_\pi^2 = (m_u + m_d) B_0 \quad \text{(GOR relation)} \]

• Fourth order term

\[ \mathcal{L}^{(4)} = \frac{l_1}{4} (\text{Tr}[\partial_{\mu} U^{\dagger} \partial^\mu U])^2 + \frac{l_2}{4} \text{Tr}[\partial_{\mu} U^{\dagger} \partial_{\nu} U] \text{Tr}[\partial^\nu U^{\dagger} \partial^\mu U] \]

\[ + \frac{l_3}{4} B_0^2 (\text{Tr}[m(U + U^{\dagger})])^2 + \frac{l_4}{4} B_0 \text{Tr}[\partial_{\mu} U^{\dagger} \partial^\mu U] \text{Tr}[m(U + U^{\dagger})] + \ldots \]

Collect all Feynman diagrams generated by \( \mathcal{L}_{\text{eff}} \). Classify all terms according to powers of a variable \( Q \) which stands generically for three-momentum or energy of the Goldstone bosons, or for the pion mass \( m_\pi \). The small expansion parameter is \( Q / 4\pi f_\pi \). Loops are subject to dimensional regularisation and renormalisation.

The unknown coefficients are fixed phenomenologically (no real matching of the EFT to QCD)

- \( \pi\pi \), KK (SU(3) extension) scatterings \( \rightarrow \) many successes
- Unitarized \( \chi PT \)
- Confirmation of the strong condensate scenario (validity of GOR)
Inclusion of baryons

\[ \chi_{\pm} = u^\dagger \chi u^\dagger \pm u^\dagger u, \quad u^2 = U \]

\[ \chi = 2BM \]

\[ L^{(1)}_N = \bar{\Psi} (i\gamma^\mu D^\mu - M_0) \Psi + \frac{1}{2} g_A \bar{\Psi} \gamma^\mu \gamma^5 u^\mu \Psi, \]

\[ L^{(2)}_N = c_1 \text{Tr}(\chi_+) \bar{\Psi} \Psi - \frac{c_2}{4M_0^2} \text{Tr}(u^\mu u_{\nu}) (\bar{\Psi} D^\mu D^\nu \Psi + \text{h.c.}) + \frac{c_3}{2} \text{Tr}(u^\mu u^\nu) \bar{\Psi} \Psi + \ldots \]

\[ g_A = 1.27, \quad c_1 \text{ related to } \sigma_N = 50 \text{ MeV which is the pion-nucleon sigma term} \]

Many successful applications

- Threshold pion photo electroproduction, Compton scattering on nucleon
- Pion-nucleon scattering
- KN scattering, coupling to resonances via unitarized coupled channels
- NN interaction

Limitations of ChiPT

- The structure and the size of the nucleon is hidden: relative role of the pion cloud vs scalar field not known
- The scalar radial field is frozen: ChiPT has little to say for mass evolution, in-medium scalar polarization of the nucleon, mechanisms for chiral restoration
- Unitarization (account of resonances) by hand destroying power counting

\[ \sigma_N = m_q \frac{\partial M_N}{\partial m_q} = \langle N | m_q (\bar{u}u + \bar{d}d) | N \rangle \]

\[ M_N = M_0 + \sigma_N \]
3- Chiral restoration

a-Lattice results

Hadronic matter heated or compressed quarks «percolate» / liberated

Sudden change of energy density
Sharp decrease of the quark condensate $\langle \bar{q}q \rangle$

What are the mechanism of chiral restoration, interplay with confinement?
What about finite density (finite chemical potential, low T)?
b- Partial restoration of chiral symmetry

• Dropping of the quark condensate

The quark condensate, i.e., the scalar density of the QCD vacuum is negative. The hadrons have a positive scalar density originating from valence constituent quarks (scalar field) and pion cloud

\[
\langle \bar{q} q \rangle = \langle \bar{q} q \rangle_{\text{vac}} + \sum_h \rho_h Q_S^h \quad \text{with} \quad Q_S^h = \int d^3 r \langle \bar{\psi} \psi(r) \rangle_h
\]

Introduce the sigma commutator of the hadron

\[
\sigma_h = m Q_S = \langle h | H_{\chi SB} | h \rangle = \langle h | [Q_k^5, [Q_k^5, H]] | h \rangle = m \frac{\partial M_h}{\partial m}
\]

Assuming the GOR valid

\[
R = \frac{\langle \bar{q} q \rangle (\rho, T)}{\langle \bar{q} q \rangle} \approx 1 - \sum_h \frac{\rho_{sh} \Sigma_h}{f_{\pi}^2 m_{\pi}^2}
\]

• The quark condensate from the equation of state

\[
\Omega(V, T, \mu_B) = -T \ln Z = -T \ln \left( T \mathbb{Tr} \left[ e^{-\beta(H_{\text{QCD}} - \mu_B N_B)} \right] \right) = \Omega_{\text{vac}} - V P(T, \mu_B) \equiv V \omega(T, \mu_B)
\]

Feynman-Hellmann theorem

\[
\langle \langle \bar{q} q \rangle \rangle(T, \mu_B) = \frac{1}{2} \left( \frac{\partial \omega}{\partial \mu_B} \right)_{\mu_B} = \langle \bar{q} q \rangle_{\text{vac}} - \frac{1}{2} \left( \frac{\partial P}{\partial \mu_B} \right)_{\mu_B}
\]
c- Chiral restoration and hadron structure

\[ \frac{\langle \bar{q}q \rangle \langle \mu_B, T \rangle}{\langle \bar{q}q \rangle_{\text{vac}}} = 1 - \sum_h \frac{\rho_s(\mu_B, T) \sigma_h}{f_\pi^2 m_\pi^2} \]

Only the lightest hadrons contribute, heavy hadrons (with large momenta valence quarks) decouple from the condensate.

- Leading order in \( T \) (pion gas)
- Dilute nuclear matter (nucleons)

\[ \frac{\langle \bar{q}q \rangle \langle \rho \rangle}{\langle \bar{q}q \rangle_{\text{vac}}} \sim 1 - 0.35 \frac{\rho}{\rho_0} - \frac{T^2}{8 f_\pi^2} \]

\[ \sigma_h = m \frac{\partial M_h}{\partial m} = m_\pi^2 \frac{\partial M_h}{\partial m_\pi^2} \]

"Data": lattice [Bowman et al '02]
Curve: Instanton Model
[Diakonov+Petrov '85, Shuryak]
Extrapolation of lattice data

\( M_N, \sigma_N \) obtainable in principle from lattice, but lattice data available only for \( m_\pi > 400 \) MeV → Use ChiPT to extrapolate

But extrapolation of ChiPT at order \( m^3 \pi \) fails:

1. Use Higher order ChiPT (Procura et al)

2. Use chiral model to extrapolate lattice data (Thomas et al)

\[
M_N^{(3)} = M_0 - 4c_1 m_\pi^2 - \frac{3g_A^2}{32\pi f_\pi^2} m_\pi^3
\]

The specific contribution of the pion cloud is very important and depends on a scale: the nucleon size (hidden in \( \chi PT \))

\[ \sigma_N \approx 50 MeV, \] half of it from the pion cloud
Annex: model calculation of the pion-nucleon sigma commutator

Linear sigma model or bosonized NJL + confinement
- $g_S$: scalar coupling constant
- $v(q)$: pion-nucleon form factor

\[ H_{\chi SB} = \int d^3 r \left( -f_\pi m_\pi^2 s(r) + \frac{1}{2} m_\pi^2 \phi^2(r) \right) \]

\[ \sigma_N = \langle N | H_{\chi SB} | N \rangle = \sigma_N^{(\sigma)} + \sigma_N^{(\pi)} \]

\[ \sigma_N^{(\sigma)} = f_\pi g_S \frac{m_\pi^2}{m_\sigma^2} \approx 29 \text{ MeV} \]

\[ \sigma_N^{(\pi)} = \frac{3}{2} \left( \frac{g_A}{2f_\pi} \right)^2 m_\pi^2 \frac{d q}{(2\pi)^3} \int \frac{d q}{2\omega_q^2} \left[ \frac{q^2 v^2(q)}{2\omega_q^2} \left( \frac{1}{\omega_q \omega_q + \epsilon_q} + \frac{1}{(\omega_q + \epsilon_q)^2} + \frac{4 R_{N\Delta}^2}{9} \left( \frac{1}{\omega_q \omega_q + \epsilon_{\Delta q}} + \frac{1}{(\omega_q + \epsilon_{\Delta q})^2} \right) \right] \approx 21 \text{ MeV} \]
d- Fluctuations of the condensate and chiral susceptibilities

• **Scalar susceptibility**: from the scalar correlator *i.e. the correlator of the scalar quark density fluctuations*

\[
\chi_s = \frac{\partial \langle \langle \bar{q} q \rangle \rangle}{\partial m} = 2 \int dt \, dr^2 \langle \langle \delta \bar{q} q(0,0), \delta \bar{q} q(r^2, t) \rangle \rangle
\]

(Obtainable from the EOS)

• **Compare susceptibilities associated with chiral partners**

Scalar (sigma): \( \bar{q} q \) \hspace{1cm} Pseudoscalar (pion): \( \bar{q} i\gamma_5 \frac{\tau_\alpha}{2} q \)

**SCALAR SUSCEPTIBILITY**

\[
\chi_s = \frac{\partial \langle \bar{q} q \rangle}{\partial m_q} = 2 \int dt \, dr' \Theta(t - t') \langle -i \left[ \bar{q} q(0), \bar{q} q(r', t') \right] \rangle
\]

\[
\chi_s = \left( \frac{\partial^2 \omega}{\partial m_q^2} \right)_{\mu} = Re G_S(\omega = 0, q \to 0) = \int_0^\infty d\omega \left( -\frac{2}{\pi\omega} \right) Im G_S(\omega, q = 0)
\]

At finite density a strong contribution of low energy nuclear excitations is expected

**PSEUDOSCALAR SUSCEPTIBILITY**

\[
\chi_{PS} = 2 \int dt \, dr' \Theta(t - t') \langle -i \left[ \bar{q} i\gamma_5 \frac{\tau_\alpha}{2} q(0), \bar{q} i\gamma_5 \frac{\tau_\alpha}{2} q(r', t') \right] \rangle = \frac{\langle \bar{q} q \rangle(\rho)}{m_q}
\]
Thermal susceptibility on Lattice (Karsch)

Finite density: effective chiral theory (M. Ericson, G. C.)

Chiral Restoration: $\chi_S \rightarrow \chi_{PS}$
Medium effects as precursors of chiral restoration

Bottom to top attitude: follow the evolution of the hadron spectral functions and relate it to the evolution of the QCD properties (condensates)

**QCD sum rules:** relate the hadron spectral function \((\rho, \omega, \phi)\) to quarks (and gluons) condensates

Current-current correlation function from its spectral representation

\[
\Pi(q^2) = \frac{i}{3} \int d^4x \, e^{iq.x} < 0|T(J_\mu(x), J_\mu(0))|0> = \Pi(0) + q^2 \int_0^\infty \frac{ds}{s} \frac{(-\frac{1}{\pi})}{q^2 - s + i\eta} Im \Pi(s)
\]

Currents with quantum numbers of the hadron

\[
J_\rho^\mu = \frac{1}{2} (\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d), \quad J_\omega^\mu = \frac{1}{6} (\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d), \quad J_\phi^\mu = -\frac{1}{3} \bar{s}\gamma^\mu s
\]

For large space-like momenta \((Q^2 = -q^2 > 0)\), use « OPE » (FT of Taylor expansion around \(x=0\))

\[
\Pi(q^2 = -Q^2) = \int_0^\infty \frac{ds}{s} \frac{(-\frac{1}{\pi})}{s + Q^2} Im \Pi(s)
\]

\[
= \frac{dV}{12 \pi^2} \left[ -c_0 \ln \left( \frac{Q^2}{\mu^2} \right) + \frac{c_1}{Q^2} + \frac{c_2}{Q^4} + \frac{c_3}{Q^6} + \ldots \right]
\]

Problem: OPE valid at large \(Q^2\) where many resonances contribute to the dispersive integrals
The trick: Borel transform: $Q^2 \rightarrow M_B$

\[
\frac{1}{M_B^2} \left[ \Pi(0) + \int \frac{d\omega^2}{\omega^2} e^{-\omega^2/M_B^2} \left( -\frac{1}{\pi} \right) I_m \Pi(\omega, 0) \right] = \frac{dV}{12 \pi^2} \left[ c_0 + \frac{c_1(\rho)}{M_B^2} + \frac{c_2(\rho)}{M_B^4} + \frac{c_3(\rho)}{2 M_B^6} + \ldots \right]
\]

Choice for $M_B$: convergence of the integral and OPE: $1 \text{ GeV} < M_B < 1.5 \text{ GeV}$

**VACUUM:**

\[
R(s) = -\frac{12\pi}{s} I_m \Pi(s) = \rho \nu DM \left( s \right) + dV \left( 1 + \frac{\alpha_s}{\pi} \right) \Theta(s - s_V)
\]

\[
c_0 = 1 + \frac{\alpha_s(Q^2)}{\pi}, \quad c_1 = -3(m_u^2 + m_d^2),
\]

\[
c_2 = \frac{\pi^2}{3} < G \cdot G > + 4\pi^2(m_u < \bar{u}u > + m_d < \bar{d}d >)
\]

\[c_3 \sim \text{“Condensats à quatre quarks”} \sim < (\bar{q}q)^2 >
\]

Pole ansatz: $\rho \nu DM \left( s \right) = F_V \delta(s - m_V^2)$.

\[
m_{\rho,\omega} = 0.77 \text{ GeV}, \quad m_\Phi = 1.02 \text{ GeV}
\]

**MEDIUM (finite density)**

\[
\Pi^{\mu\nu}(q) = -i \int d^4x e^{iq\cdot x} < A(\rho) | T(J^\mu(x), J^\nu(0)) | A(\rho) >
\]

\[
\frac{1}{M_B^2} \left[ \Pi(0) + \int \frac{d\omega^2}{\omega^2} e^{-\omega^2/M_B^2} \left( -\frac{1}{\pi} \right) I_m \Pi(\omega, 0) \right] = \frac{dV}{12 \pi^2} \left[ c_0 + \frac{c_1(\rho)}{M_B^2} + \frac{c_2(\rho)}{M_B^4} + \frac{c_3(\rho)}{2 M_B^6} + \ldots \right]
\]

Enter the evolution of the condensate: Main uncertainty is $c_3$: $< (\bar{q}q)^2 > \sim < \bar{q}q >^2$
**ω meson:** QCD sum rule consistent with a pole ansatz

\[ \frac{m_\omega^k}{m_\omega} = 1 - 0.18 \frac{\rho}{\rho_0}, \]

**ρ meson:** loses its quasiparticle status: a simple pole ansatz leads to an erroneous dropping mass scenario! (Klingl et al)
Axial-vector mixing and Weinberg sum rules

Vector and axial-vector correlators

\[ \Pi_{VV}^{\mu\nu}(q) = -i \int d^4 x e^{iq \cdot x} \ll T (\mathcal{V}_k^{\mu}(x), \mathcal{V}_k^{\nu}(0)) \gg \]
\[ \Pi_{AA}^{\mu\nu}(q) = -i \int d^4 x e^{iq \cdot x} \ll T (\mathcal{A}_k^{\mu}(x), \mathcal{A}_k^{\nu}(0)) \gg \]

and the corresponding spectral functions

\[ \frac{1}{\pi} \text{Im} \Pi_{VV}^{\mu\nu}(q; T = 0) = -(q^2 g^{\mu\nu} - q^\mu q^\nu) \rho_V(q^2) \]
\[ \frac{1}{\pi} \text{Im} \Pi_{AA}^{\mu\nu}(q; T = 0) = q^\mu q^\nu f_\pi^2 \delta(q^2 - m_\pi^2) - (q^2 g^{\mu\nu} - q^\mu q^\nu) \rho_A(q^2) \]

From chiral symmetry alone to order $T^2$ (chiral limit), the only medium effect is the « mixing » of the correlators (no mass shift)

\[ \Pi_{VV}^{\mu\nu}(q; T) = (1 - \epsilon) \Pi_{VV}^{\mu\nu}(q; T = 0) + \epsilon \Pi_{AA}^{\mu\nu}(q; T = 0) \]
\[ \Pi_{AA}^{\mu\nu}(q; T) = (1 - \epsilon) \Pi_{AA}^{\mu\nu}(q; T = 0) + \epsilon \Pi_{VV}^{\mu\nu}(q; T = 0) \]

The mixing is driven by the pion scalar density

\[ \epsilon = \frac{T^2}{6 f_\pi^2} = \frac{2}{f_\pi^2} \int \frac{d \mathbf{k}}{(2\pi)^3} \frac{n(\omega_k)}{\omega_k} = \frac{2}{3} \ll \Phi^2 \gg \]
\[ f_\pi^*(T) = \sqrt{1 - \epsilon} \simeq 1 - \frac{T^2}{12 f_\pi^2} = 1 - \frac{1}{3} \ll \Phi^2 \gg \]

This axial-vector mixing driven by in-medium pion loop effects can be generalized for finite density and is at the heart of the interpretation of the dilepton data (NA60)
Chiral symmetry breaking: a low energy long range phenomena

Weinberg sum rules

\[ \int_0^\infty ds \left( \rho_V(s) - \rho_A(s) \right) = f_\pi^2, \quad \int_0^\infty ds s \left( \rho_V(s) - \rho_A(s) \right) = 0 \]

In-medium

\[ \int_0^\infty d\omega^2 \left[ \left( -\frac{\text{Im} \Pi_V(\omega, q = 0)}{\pi \omega^2} \right) - \left( -\frac{\text{Im} \Pi_A(\omega, q = 0)}{\pi \omega^2} \right) \right] = 0 \]

\[ \int_0^\infty d\omega^2 \omega^2 \left[ \left( -\frac{\text{Im} \Pi_V(\omega, q = 0)}{\pi \omega^2} \right) - \left( -\frac{\text{Im} \Pi_A(\omega, q = 0)}{\pi \omega^2} \right) \right] = 0 \]

Pole ansatz

\[ -\frac{\text{Im} \Pi_V(\omega, q = 0)}{\pi \omega^2} = \frac{\rho^4}{g_\rho^2} \frac{Z_\rho(T)}{\omega^2} \delta(\omega^2 - m^*_\rho^2(T)) \]

\[ -\frac{\text{Im} \Pi_A(\omega, q = 0)}{\pi \omega^2} = \frac{\rho^4}{g_\rho^2} \frac{Z_A(T)}{\omega^2} \delta(\omega^2 - m^*_A^2(T)) + f_\pi^2(T) \delta(\omega^2) \]

Vacuum:

\[ \frac{m_\rho^4}{g_\rho^2} = \frac{m_A^4}{g_A^2}, \quad m_\rho^2 = a g_\rho^2 f_\pi^2 \text{ avec } a = \left( 1 - \frac{m_\rho^2}{m_A^2} \right)^{-1} \]

In-medium

\[ \frac{f_\pi^2(T)}{f_\pi^2} = a Z_\rho(T) \left( \frac{m_\rho^2}{m^*_\rho^2(T)} - \frac{m_\rho^2}{m^*_A^2(T)} \right) \]

The centroids \( m_\rho^* (T) \) and \( m_A^* (t) \) becomes identical at full restoration.
But we do not know the scenario!

**Axialvector / Vector in Vacuum**

\[
\text{\begin{itemize}
\item \( V [\tau \to 2\pi \nu_\tau] \)
\item \( A [\tau \to (2n+1)\pi \nu_\tau] \)
\end{itemize}}
\]

\[
\Im \frac{\Pi_{\text{em}}}{\Pi_{\text{em},\text{cont}}} \approx \begin{cases}
\rho(770) + \text{cont.} \\
a_1(1260) + \text{cont.}
\end{cases}
\]

\[
\text{Mass}
\]

**Low-Mass Dilepton Rate:**

\[
\frac{dN_{ee}}{d^4x d^4q} = -\alpha_s^2 f^B(T) \text{Im} \Pi_{\text{em}} \sim [\text{Im} D_{\rho} + \text{Im} D_{\omega}/10 + \text{Im} D_{\phi}/5]
\]

\[
\rho - \text{meson dominated!}
\]

**Axialvector Channel:**

\[
\pi^\pm \gamma \text{ invariant mass-spectra} \sim \text{Im} D_{a_1}(M) ?!
\]
**f- Chiral effective theory**  \( (G.C \ et \ al) \)

\[
\sigma + i \vec{\tau} \cdot \vec{\pi} = S U = (f_\pi + s) \exp \left[ i \frac{\vec{\pi} \cdot \vec{\phi}}{f_\pi} G \left( \frac{\phi^2}{f_\pi^2} \right) \right]
\]

\[
\frac{\langle \bar{q}q \rangle_{\text{medium}}}{\langle \bar{q}q \rangle_{\text{vacuum}}} = 1 - \frac{\langle \phi^2_\pi(\rho) \rangle}{2 f_\pi^2} - \frac{|\langle s(\rho) \rangle|}{f_\pi}
\]

- Pionic fluctuation: \( \langle \phi^2_\pi \rangle \iff \text{A-V mixing} \)
- Shrinking of the chiral radius (relativistic Walecka theory, QHD): \( \langle S \rangle = f_\pi + \langle s \rangle \) decreases \( \iff \) Dropping of the masses

**Effective lagrangian**

Mean field (\( S^+ \) \( \pi N \) piece+ Pion with derivative couplings to \( S \))

\[
\mathcal{L} = i \bar{N} \gamma^\mu \partial_\mu N - M_N^*(s) \bar{N} N + \frac{1}{2} \partial^\mu s \partial_\mu s - \frac{m^2_{\pi^2} - m^2_{\pi^2}}{8 f_\pi^2} \left( s^2 + 2 f_\pi s + \frac{2 f^2 m^2_{\pi^2}}{m^2_{\pi^2} - m^2_{\pi^2}} \right)^2 + \mathcal{L}_\omega + \mathcal{L}_{\pi NN}^\text{pwave} + \mathcal{L}_{\pi \pi} + \mathcal{L}_{\chi SB} + \mathcal{L}_{WT}
\]

with \( U = \xi^2 = \exp \left[ i \frac{\vec{\tau} \cdot \vec{\phi}}{f_\pi} G \left( \frac{\phi^2}{f_\pi^2} \right) \right] \), \( u_\mu = i \xi^\dagger \partial_\mu U \xi^\dagger \)
II-Some Examples of medium effects:
1 - Pions, \( \sigma \) and \( \rho \) in nuclear and hadronic matter

**a - Pion mass in nuclear matter**

S-wave pion optical potential

\[
2m_{\pi}U_s(r) = -4\pi \left( 1 + \frac{m_{\pi}}{M} \right) \left[ (b_0)_{\text{eff}} \rho + b_1(\rho_n - \rho_p) \right] - 4\pi \left( 1 + \frac{m_{\pi}}{2M} \right) B_0 \rho^2
\]

**Recent data on \(^{205,207}\text{Pb}: deeply bound 1s and 2p states**

To reproduce the data one needs an increase of \( b_1 \)

\[
\left( \frac{b_1}{b_1^0} \right)^{\text{exp}} = 0.78^{+0.13}_{-0.09}
\]

Compatible with \( \Delta M_{\pi^-}^{\text{exp}} = 23 \sim 27 \text{MeV} \) (Standard 16 MeV)

(at density \( \rho = 0.6\rho_0 \) probed by the \( \pi^- \))

In the center of Pb

**Weinberg-Tomozawa amplitude**

\[
\mathcal{L}_{\text{WT}} = -\frac{1}{4 f_{\pi}^2} \bar{N} \gamma^\mu \vec{\tau} \cdot \vec{\phi} \times \partial_\mu \vec{\phi} N
\]

\[
\rightarrow \left( 1 + \frac{m_{\pi}}{M_N} \right) b_1 = \frac{m_{\pi}}{8\pi f_{\pi}^2}
\]

**GOR relation**

\[
\frac{b_1^*}{b_1} = \frac{f_{\pi}^2}{f_{\pi}^{*2}} = \frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle^*} \approx 1 + \frac{\sigma N \rho}{f_{\pi}^2 m_{\pi}^2} \approx 1 + 0.40 \frac{\rho}{\rho_0}
\]

A signal for chiral symmetry restoration?
\[ \mathcal{L} = i \bar{N} \gamma^\mu \partial_\mu N - M_N^*(s) \bar{N} N + \frac{1}{2} \partial^\mu s \partial_\mu s - \frac{m_s^2 - m_N^2}{8 f^2} \left( s^2 + 2 f_\pi s + \frac{2 f_\pi^2 m_N^2}{m_s^2 - m_N^2} \right)^2 + \mathcal{L}_\omega + \mathcal{L}_{\text{wave}} \]

\[ + \mathcal{L}_{\pi\pi} + \mathcal{L}_{\chi_{SB}} + \mathcal{L}_{\text{WT}} \quad \text{with} \quad U = \xi^2 = \exp \left[ \frac{i}{f_\pi} \frac{\phi}{f_\pi} \right], \quad u_\mu = i \xi^\dagger \partial_\mu U \xi^\dagger \]

• Nuclear saturation: \( M_N^*(s) = M_N \left( 1 + \frac{s}{f_\pi} + \frac{s^2}{f_\pi^2} + \frac{1}{3} \frac{s^3}{f_\pi^3} \right), \quad m_\sigma = 750 \text{MeV} \)

• 2\(\pi\), 4\(\pi\), ..., \(\pi N\) Lagrangian:
  
  \[ \mathcal{L}_{\pi\pi} = \frac{1}{4} (f_\pi + s)^2 \text{Tr} \partial^\mu U \partial_\mu U^\dagger \]

  \[ \mathcal{L}_{\chi_{SB}} = f_\pi m_\pi^2 (f_\pi + s) \cos \left( \frac{\phi}{f_\pi} G \left( \frac{\phi^2}{f_\pi^2} \right) \right) \]

  \[ \Delta \mathcal{L}_{\pi N}^{(2)} = c_3 \bar{N} (u \cdot u) N + c_2 \bar{N} (v \cdot u)^2 N \]

• Isovector \(\pi N\) Lagrangian:
  
  \[ \mathcal{L}_{\text{WT}} = \bar{N} \gamma_\mu \frac{i}{2} (\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi) N \]

• Representation dependence: \( G(X^2) = 1 + \alpha X^2 + ... \)
Chiral perturbation theory to order $O(Q^6)$

Pion self-energy

\[
\Pi(\omega) = \rho \left\{ - \left( \sigma_N^2 f^2 \right) + \omega^2 \left( \frac{g_1^2}{4M_N} f^2 - 2 \frac{c_2 + c_3}{f^2} \right) - \frac{2\beta}{3} \left( \omega^2 - m^2 \right) \frac{\Sigma^{(\pi)}_{\text{LNAC}}}{f^2 m^2} \right\}
\]

Wave function renormalisation but depends on the representation

$\beta = 1 + 10(\alpha - 1/6) = 0$

(Kolomeitsev, Kaiser, Weise)

\[
\left( \frac{b^*_1}{b_1} \right)_{\text{CHIPPT1}} = Z_{\pi} \simeq 1 + \frac{\sigma N \rho}{f^2 m^2} - \frac{4}{3} \frac{\Sigma^{(\pi)}_{\text{LNAC}}}{f^2 m^2} \rho - \frac{2\beta}{3} \frac{\Sigma^{(\pi)}_{\text{LNAC}}}{f^2 m^2} \rho \simeq 1 + 0.53 \frac{\rho}{\rho_0}
\]

Chiral effective theory

- Explicit incorporation of the radial mode
- Explicit incorporation of the full pion loop depending on the nucleon size

\[
\Pi(\omega) = \rho \left\{ - \left( \sigma_N^2 f^2 \right) + \omega^2 \left( \frac{g_1^2}{4M_N} f^2 - 2 \frac{c_2 + c_3 - \Sigma^{(\pi)}_{\text{LNAC}}}{f^2} \right) - \frac{2\beta}{3} \left( \omega^2 - m^2 \right) \frac{\Sigma^{(\pi)}_{\text{LNAC}}}{f^2 m^2} \right\}
\]

$Z_{\pi} \simeq 1 + \frac{\sigma N \rho}{f^2 m^2} - \frac{4}{3} \frac{\Sigma^{(\pi)}_{\text{LNAC}}}{f^2 m^2} \rho - \frac{2\beta}{3} \frac{\Sigma^{(\pi)}_{\text{LNAC}}}{f^2 m^2} \rho$

(G.C., Ericson, Oertel)
But vertex correction needed to eliminate the representation dependence and depending on the pion scalar density

\[ \mathcal{L}_{WT}^{\text{eff}} = -\frac{1}{4 f_\pi^2} \left( 1 + \frac{20}{3} \left( \alpha - \frac{1}{24} \right) \frac{\Sigma_N^{(\pi)}}{f_\pi^2 m_\pi^2} \right) \times \bar{N} \gamma^\mu \pi \cdot (\phi \times \partial_\mu \phi) N \]

\[ \frac{b_1^*}{b_1} = 1 + \frac{\sigma N \rho}{f_\pi N m_\pi^2} - \frac{7}{6} \frac{\Sigma_N^{(\pi)}}{f_\pi^2 m_\pi^2} \rho \simeq 1 + 0.18 \frac{\rho}{\rho_0} \]

And rescattering in presence of Pauli correlations, actually energy dependent

\[ \rho \Sigma_N^{(\pi)} = \frac{\langle \phi_{\pi}^2 \rangle}{2 m_\pi^2} \]

\[ \delta \Pi^{\text{esc}} = -4\pi \delta b_0^{\text{esc}} \rho \frac{\omega^2}{m_\pi^2} \]

\[ \frac{\delta b_1^*}{b_1} = \frac{3}{16\pi} \frac{k_{\text{F}}^4 \rho}{f_\pi^4} \simeq 0.26 \left( \frac{\rho}{\rho_0} \right)^{4/3} \]

\[ \frac{b_1^*}{b_1} = 1.19 \quad \text{at} \quad \rho = 0.5 \rho_0 \]

- In-medium chiral dynamics, CERTAINLY YES
- Chiral restoration: related to (through the \( \langle \phi_{\pi}^2 \rangle \))
Evolution of masses and coupling constants at moderate densities

\[ \frac{\langle \langle qg \rangle \rangle^*}{\langle qg \rangle_{\text{vac}}} = 1 - \frac{\rho \Sigma_N^{(\pi)}}{f_\pi^2 m_\pi^2} - \frac{\rho \Sigma_N^{(\pi)}}{f_\pi^2 m_\pi^2} \]

\[ \frac{M_N^*}{M_N} = 1 - \frac{\rho \Sigma_N^{(\pi)}}{f_\pi^2 m_\pi^2} \]

\[ \frac{m_0^*}{m_0} = 1 - \frac{3}{2} \frac{\rho \Sigma_N^{(\pi)}}{f_\pi^2 m_\pi^2} \]

\[ \frac{f_\pi^*}{f_\pi} = 1 - \frac{\rho \Sigma_N^{(\pi)}}{f_\pi^2 m_\pi^2} - \frac{2}{3} \frac{\rho \Sigma_N^{(\pi)}}{f_\pi^2 m_\pi^2} \]

\[ \frac{g_A^*}{g_A} = 1 - \frac{2 g_A - 1}{g_A} \frac{\rho \Sigma_N^{(\pi)}}{f_\pi^2 m_\pi^2} - \frac{2}{3} \frac{\rho \Sigma_N^{(\pi)}}{f_\pi^2 m_\pi^2} \]

\[ \frac{m_\pi^*}{m_\pi} = 1 + \frac{1}{2} \frac{\rho \Sigma_N^{(\pi)}}{f_\pi^2 m_\pi^2} + \frac{1}{6} \frac{\rho \Sigma_N^{(\pi)}}{f_\pi^2 m_\pi^2} + c_2, c_3 + \ldots 1 \]

\[ \sigma_N = \sigma_N^{(\pi)} + \sigma_N^{(\sigma)} \]

\[ \frac{\Sigma_N^{(8)}}{f_\pi^2 m_\pi^2} \rho \approx 0.21 \frac{\rho}{\rho_0}, \quad \frac{\Sigma_N^{(\pi)}}{f_\pi^2 m_\pi^2} \rho \approx 0.18 \frac{\rho}{\rho_0} \]

\[ \sigma_N^{(\pi)} = \frac{3}{2} \left( \frac{g_A}{2 f_\pi} \right)^2 m_\pi^2 \int \frac{dq}{(2\pi)^3} \frac{q^2 v^2(q)}{2\omega_q^2} \left[ \frac{1}{\omega_q} \frac{1}{\omega_q + \epsilon_q} + \frac{1}{(\omega_q + \epsilon_q)^2} \right] \]

\[ + \frac{4 R_N^2}{9} \left( \frac{1}{\omega_q} \frac{1}{\omega_q + \epsilon_{\Delta q}} + \frac{1}{(\omega_q + \epsilon_{\Delta q})^2} \right) \]
b- In-medium modified pion dispersion relation

- Pion-nucleon p-wave coupling

\[ H_{\pi NN} = -\int dx \frac{g_A}{f_\pi} \overline{N} \gamma^\mu \gamma_5 \frac{\tau}{2} \partial_\mu \Phi N \simeq -\int dx \frac{g_A}{2f_\pi} N^\dagger \sigma \cdot \nabla \Phi \cdot \overline{\tau} N \]

Coupling to \( \Delta \) states

\[ \sigma \cdot q \, T_j \rightarrow (g_{\pi N\Delta}/g_{\pi NN}) S \cdot q \, T_j \]

- In-medium pion propagator

\[ D(\omega, k) = \left( \omega^2 - \omega_k^2 - S(\omega, k) \right)^{-1} \]

\[ S(\omega, k) = k^2 \overline{\Pi}_0(\omega, k), \quad \overline{\Pi}_0(\omega, k) = \frac{\Pi_0(\omega, k)}{1 - g' \Pi_0(\omega, k)} \]

At high energy the strength function is dominated by two (collective) excitations:
- The pionic branch
- The Delta branch

Not badly approximated by a two-level model

\[ D(\omega, k) = \frac{Z_1(k, \rho)}{\omega^2 - \Omega_1^2(k, \rho)} + \frac{Z_2(k, \rho)}{\omega^2 - \Omega_2^2(k, \rho)} \]
A related quantity: the longitudinal spin-isospin response

\[ R_L(\omega, k) = \left( \frac{g_{\pi NN}}{2M_N} \right)^2 v^2(k, \omega) \sum_n |n| \sum_{i=1}^{i=A} \sigma(i) \cdot k \tau_j(i) e^{ik \cdot R_i} |0 > |^2 \delta(E_n - \omega) \]

\[ = \sum_n |n| \sum_{j} L_j(k)|0 > |^2 \delta(E_n - \omega) \]

\[ \frac{1}{k^2} R_L(\omega, k) = -\frac{V}{\pi} Im \Pi_L(\omega, k) = \frac{\omega^2 - \omega_k^2}{\omega^2 - \omega_k^2 - k^2 \Pi_0(\omega, k)} \left( -\frac{V}{\pi} \right) Im \Pi_0(\omega, k) \]

(3He,T), SATURNE

No-medium effect (transverse spin)

RPA response

\[ ^{16}O \]

\[ ^{208}Pb \]

\[ \text{PROTON} \]

Shift of the strength Attributed to the pionic branch

J. Delorme
P. Guichon
Two-pion states in nuclear and hadronic matter

**Original idea:** Softening of the pion dispersion relation

> Modification of the two-pion propagator and the unitarized pion-pion interaction

> Sigma meson and rho meson channels

**Scalar-isoscalar modes**

- $A(\pi, \pi\pi)$ (CHAOS, CB), $A(\gamma, \pi\pi)(TAPS)$
  - Downwards shift of the $\pi\pi$ invariant mass distribution in the scalar-isoscalar channel $I = J = 0$
  - What is the role of
    - Chiral Dynamics
    - Chiral restoration?
Dropping of the sigma mass: associated with chiral restoration

\[ \frac{m^*_\sigma^2}{m^2_\sigma} = 1 + 3 \frac{\langle s \rangle}{f_\pi} \]

\[ \frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle_{vac}} = 1 + \frac{\langle s \rangle}{f_\pi} - \frac{\langle \phi^2_\pi \rangle}{2 f^2_\pi} \]

but constraints (conflict) with matter stability

In-medium modified pion-pion-interaction: reproduce TAPS data (Valencia group)

\[ T_{\pi\pi}(E) = V_{\pi\pi}(E) + V_{\pi\pi}(E) G(E) T_{\pi\pi}(E) \]

\[ G(E) = \int \frac{dq}{(2\pi)^3} \int \frac{idq_0}{2\pi} D_\pi(q, q_0) D_\pi(-q, E - q_0) \]

Both medium effects are related to the nuclear scalar susceptibility
**MASSES**

The sigma mass remains stable

Higher densities? Phase transition to quark matter?

**SUSCEPTIBILITIES**

**EOS**

$m_\pi = 850 \text{ MeV} \quad g_\omega = 8$

$C = 0.985 + \text{Density dependence}$

Nucleon

Sigma

Sigma + chiral dropping

$\frac{m^*}{m} \approx \frac{\partial^2 \bar{\varepsilon}}{\partial \bar{s}^2} = m_\sigma \left( 1 + \frac{3\bar{s}}{f_\pi} + \frac{3}{2} \left( \frac{\bar{s}}{f_\pi} \right)^2 \right) + \kappa NS \rho S$

The sigma mass remains stable
The full nuclear scalar susceptibility in the effective theory *(Martini et al)*

\[ \chi_S = -2 \frac{\langle q_\sigma \rangle^2}{f_\pi^2} D_\sigma(0) \]

The sigma propagator and the scalar susceptibility can be decomposed into:

- **The chiral invariant \( s \) mode with effective mass \( m_\sigma^* \) and coupled to \( p-h \) excitations; it contributes to the \( p-h \) piece of \( \chi_S \)

- **Two-pion modes**: responsible at \( E \sim 2 m_\pi \) of the downwards shift of the strength in \( 2\pi \) experiments; they contribute to the pionic piece of \( \chi_S \)

\[
D_\sigma(E) = D_s(E) + \frac{3}{2f_\pi^2} \left( 1 - 2 \frac{E^2}{E^2 - m_\pi^2} \right) \tilde{G}(E)
\]
• Vector mesons and axial-vector mixing at finite density

Rho meson propagator from the Vector dominance phenomenology

\[ J^\mu_V = \frac{m^2_V}{g_V} \mathcal{V}^\mu \]

\[ V = \rho, \omega, \Phi \]

Gauged non linear sigma model Lagrangian

\[ \mathcal{L}_{\rho h} = -g \bar{\rho}^\mu \left( \bar{\Phi} \times \partial_\mu \Phi \right) - g \bar{\tilde{N}} \gamma_\mu \tilde{\rho}^\mu \tilde{\mathcal{X}} - g \frac{g_s NN}{M_N} \bar{\tilde{N}} \gamma_\mu \gamma_5 \left( \bar{\tilde{\rho}}^\mu \times \tilde{\Phi} \right) \tilde{\mathcal{X}} + \frac{g^2}{2} \left( \bar{\tilde{\rho}}^\mu \times \tilde{\Phi} \right) \left( \tilde{\rho}^\mu \times \bar{\Phi} \right) \]

(a) Decay of the \( \rho \) into two softened collective quasi-pions

(b) From gauge invariance: kill the accumulation of strength near \( 2m_\pi \)

(c) Structure at \( \Omega_\Delta + m_\pi \sim 500 \text{ MeV} \)

G.C, P. Schuck (1992)

In-in SemiCentral

\[ \frac{dN}{dp_T} \]

G.C, P. Schuck (1992)

+ direct coupling of the rho to resonances

NA60

excess data
RW (norm.)
BR (norm.)
Vac,\rho (norm.)
cockt,\rho (dashed)
DD (dashed)
Connection with chiral restoration (G.C, J. Delorme, M. Ericson)

Axial correlator at finite density

\[
\frac{1}{f^2} A_{ij}(k) = k_i k_j D(k) + 2k_i k_j \tilde{\Pi}_0(k) D(k) + \hat{k}_i \hat{k}_j \Pi_L(k) + (\delta_{ij} - \hat{k}_i \hat{k}_j) \Pi_T(k)
\]

\[
= k_i k_j (1 + \tilde{\Pi}_0(k))^2 D(k) + \hat{k}_i \hat{k}_j \tilde{\Pi}_0(k) + (\delta_{ij} - \hat{k}_i \hat{k}_j) \Pi_T(k)
\]

Vector correlator at finite density

\[
K_{ij}(q) = \int \frac{i d^4 k_1}{(2\pi)^4} \left( \frac{1}{f^2} \left( A_{ij}(k_1) D(k_2) + A_{ij}(k_2) D(k_1) \right) \right. \\
- (1 + \tilde{\Pi}_0(k_1)) (1 + \tilde{\Pi}_0(k_2)) (k_1 k_{2j} + k_1 k_{2i}) D(k_1) D(k_2)
\]

Axial vector mixing: Vector $\leftrightarrow$ Axialvector + In-medium pion
Figure 2. $e^+e^-$ invariant mass spectra after background subtraction obtained (left) in 12 GeV proton induced reactions [6] and (right) in photonuclear reactions ($E_\gamma = 0.6$–3.8 GeV) [8].

Conflicting conclusions: KEK: 60 MeV mass shift  
Jlab: no mass shift but small broadening
2- \( \omega \) Meson in Normal Nuclear Matter

New Data for \( \gamma A \rightarrow \omega X \rightarrow \pi^0 \gamma X \): [CBELSA/TAPS '05]

- dropping \( \omega \)-mass! \((m_\omega)_{\text{med}} \approx 720\text{MeV}, (\Gamma_\omega)_{\text{med}} \approx 60\text{MeV}\)
- consistent with (some) hadronic models [Klingl et al '97]
- connection to baryon-no./chiral susceptibility? (\(\Sigma-\omega\) mixing)
refined analysis requiring recoil proton and $p-\omega$ coplanarity

D. Trnka (Gießen) priv. com.

No background subtraction!!!

$|p_\omega| < 500$ MeV/c

after LH$_2$ background subtraction

$|p_\omega| < 500$ MeV/c

$M_{\gamma\gamma}$ / MeV

$M_{\gamma\gamma}$ / MeV

$M_{\gamma\gamma}$ / MeV

$\Rightarrow$ difference in $\omega$ - line shape for proton and nuclear target confirmed;

no upward mass shift of $\omega$ meson!

$\Rightarrow$ additional structure at $\approx 600$ MeV!! (also seen for heavier targets)

fragmentation of $\omega$ strength or background ??? under investigation

preliminary
**ω meson spectral function**

*Klingl et al*

**ω → πρ dominated**

*Giessen group*

**Direct couplings to resonances**

*(S_{11}(1535))*

**Remark:** for the rho meson D_{13}(1520)
3- Kaons in dense matter

a- Motivations and basic features

- Kaons near threshold very sensitive to the EOS
- Basic low energy properties governed by the (SU(3) extension of) chiral symmetry

• Masses of kaons and antikaons in dense matter

From chiral Lagrangians (Weinberg-Tomozawa):

Isospin breaking in nuclear matter

\[ \Delta m_{\pi^+} = -\Delta m_{\pi^-} = \frac{1}{4f^2_{\pi}}(\rho_p - \rho_n) = \frac{g^2_{\rho}}{2m^2_V} (\rho_p - \rho_n) \]

SU(3) flavor symmetry breaking at low T, large \( \mu_B \)

\[ \Delta m_{K^+} = +\frac{1}{2} \frac{1}{4f^2_{\pi}} \left( 3(\rho_p + \rho_n) + (\rho_p - \rho_n) \right) - \frac{\Sigma_{KN}}{2m_K f^2_{\pi}} \rho \]

\[ \Delta m_{K^-} = -\frac{1}{2} \frac{1}{4f^2_{\pi}} \left( 3(\rho_p + \rho_n) + (\rho_p - \rho_n) \right) - \frac{\Sigma_{KN}}{2m_K f^2_{\pi}} \rho \]
Some experimental facts

From SIS (KaoS, FOPI)
- Absence of in-plan flow of $K^+$
- Strong azimuthal emission of $K^+$; opposite to $K^-$
- Near threshold or subthreshold production of $K^+$ and $K^-$ \( \frac{K^-}{K^+} \approx 1 \gg 0.1 \) (elem. processes)

In-medium attraction for the $K^-$
In-medium repulsion for the $K^+$

Semi-central Ni+Ni 1.93AGeV
**b- Kaons (K⁺)**

- **Flow variables:**
  - Need a in-medium repulsive Potential \((Ko\ et\ al)\)

- **K⁺ spectra, excitation functions near or sub-threshold**
  - K⁺ produced relatively early
  - Secondary processes important
  - Repulsive KN potential seen in absolute yields

- **Sensitivity to the EOS**

  In heavy system (Au+Au) the sensitivity to The EOS decreases with KN potential but still survives in Au+Au/C+C \((C.\ Fuchs)\)
**c- Antikaons (K⁻)**

- **Weinberg-Tomozawa**: $K$-$N$ attractive
- **Scattering lengths**: $K$-$N$ repulsive in vacuum
- **Kaonic atoms**: $K$-$N$ attractive strongly attractive in-medium

**WHY? $\Lambda(1405)$ as a $K$-$N$ bound state:**
Transform attraction into repulsion

- **In-medium antikaons nucleons interaction**

  - **Pauli blocking**: $\Lambda(1405)$ « dissolves » in the medium: Self-consistent treatment (V. Koch)
  - **Wass et al**
  - **Ramos et al**
  - **Lutz et al**

$\Lambda(1405)$ does not move but broadens $K$-$N$ potential momentum dependent little attraction in HIC

**Chiral dynamics generates a Strong reshaping and broadening of the $K$-spectral function**
Low momentum $K^-$
Upper branch: $\Lambda(1405)$-hole
Lower branch: $K^-$ pole
Melting of the $K^-$ with resonances

High momentum $K^-$
No $K^-N$ potential!

Enhanced production of $K^-$ through medium effects in secondary processes (again through the 1405)

(Schaffner-Bielich et al)
A possible scenario (H. Oeschler) supported by transport codes

1- Production of hyperon and kaons
\[ BB \rightarrow NY(\Lambda)K \]
Strangeness conservation
\[ N(Y) = N(K^+) \]

2- \( K^- \) produced by secondary processes: Fast reaction
\[ \Pi Y \leftrightarrow K^-N \]
Relative chemical equilibrium between hyperons anti-kaons
(not very sensitive to the cross-sections)
\[ N(K^-) \text{ determined by } N(Y) \]
\[ N(Y) \gg N(K^-) \]

\( K^+/K^- \) determined

\( K^- \) production more sensitive to the \( K^+N \) potential than \( K^-N \) potential
4-Hidden and open charm

- Charmonia \((J/\psi)\) Sensitive to the gluon condensate
  \[ \Delta m \leq 10\,\text{MeV} \]

- Open charm \(D(c\bar{q})/D(q\bar{c})\)
  Light quark \(q\) fluctuating around a heavy color source: sensitive to the quark condensate (QCD sum rules):
  \[ \Delta m = -50 (\rho/\rho_0)\,\text{MeV} \]
  - Increase of \(DD\) Production
  - Opening of \(\psi' \rightarrow DD\) Channels
  - Mechanism for the suppression of the \((J/\psi)\)

Expériences \(pA\) 

PANDA/GSI
III- In-medium hadrons and the equation of state
1 - Motivations

Nuclear many-body problem

Non-perturbative QCD

Chiral symmetry

Confinement

EOS of hadronic matter

Under extreme conditions

RELATIVISTIC MEAN FIELD

CHIRAL EFFECTIVE THEORY

HADRON SUBSTRUCTURE
2- In-medium Chiral perturbation theory

a- Three loop approximation \((Kaiser \ et \ al)\)

\[
\frac{E}{A} = \begin{array}{cccc}
\text{T} & + & + & \text{contact cut-off dependent term} \\
23.4 & +18.2 & -68.3 & +11.5
\end{array}
\]

No short-range correlations but depend on one cutoff parameter \(\Lambda\)

The bulk of the attraction comes from two-pion exchange through a contact cut-off dependent term

Gives a correct asymmetry \(a_4=34\) MeV but inclusion of \(\Delta(1232)\) improves isospin properties

But the spin-orbit not reproduced
b- Density functional theory

Relativistic mean field ($\sigma + \omega$) gives the correct (enhanced) spin-orbit

\[ U(\vec{r}) = U_V + U_S \approx (+200 - 250) \frac{\rho(\vec{r})}{\rho_0} \text{MeV} \]

\[ U_{so}(\vec{r}) = \frac{\vec{\tau} \cdot \vec{s}}{2 M_N r} \frac{d(U_V - U_S)}{dr} \approx \left( \frac{+200 + 250}{+200 - 250} \right) \frac{\vec{\tau} \cdot \vec{s}}{2 M_N r} \frac{dU}{dr} \]

\[ F_{HK}[\rho] = E_{\text{kin}}[\rho] + E_{H}[\rho] + E_{\text{xc}}[\rho] \]

Hohenberg-Kohn energy density functional \textit{(Finelli et al)}

Pion loops (ChiPT)

From scalar and vector mean fields

Constrained by low energy QCD \textit{!} (QCD sum rules and condensate evolution)

\[ \Sigma^{(0)}_S = -\frac{\sigma_N M_N}{m^2 f^2} \rho_S \]

\[ \Sigma^{(0)}_V = \frac{4(m_u + m_d) M_N}{m^2 f^2} \rho \]

\[ \Sigma^{(0)}_S = -\frac{\sigma_N}{4(m_u + m_d)} \rho_S \approx -1 \]

But the pion cloud contribution to $\sigma_N$ should be removed since it cannot contribute to the scalar self-energy, \textit{i.e.}, to the mass!
The Ground state density is build with auxiliary single particle orbitals in a self-consistent local potential built from the functional. The Kohn-Sham equations are solved in an equivalent point coupling model which reproduces the self-energies resulting from $E_H(r)$ et $E_{xc}(r)$.

Practical solution / Kohn Sham DFT

Influence of the $\Delta$

Isotopic chains

Spin-orbit spacings

Deformed nuclei
3-The role of hadron substructure (confinement)

Baryonic matter: assembly of « bags » (Y shaped color strings) in a background self-consistent scalar (radial s mode) and vector (ω,ρ) fields

\[ E = \sum_P \left( \sqrt{P^2 + M^2(s)} + g_V \omega(\bar{R}) \right) + E_{Mesons} \]

In medium baryon mass

\[ M(s) = M_B + g_S s + \frac{\kappa_{NS}}{2} s^2 + \ldots \]

**Infinite matter:**

\[ s = -\frac{g^*_S}{m^2_\sigma} \rho_S < 0 \]

\[ \omega = \frac{g_V}{m^2_\omega} \rho > 0 \]

**Saturation mechanisms:**

- \( g^*_S \) grows less rapidly than \( \rho \) (Walecka)
- \( g^*_S \) drops due to confinement (Guichon)

\[ \kappa_{NS} = \frac{\partial^2 M}{\partial^2 s} \]

QCD Scalar response (scalar polarizability) of the nucleon
Calculable in a model (MIT bag in the QMC model)
• Finite nuclei \((\text{Guichon et al})\)

### Comparison with Skyrme force

<table>
<thead>
<tr>
<th></th>
<th>(m_c) (MeV)</th>
<th>(t_0) (fm(^4))</th>
<th>(t_1) (fm(^4))</th>
<th>(t_2) (fm(^4))</th>
<th>(t_3) (fm(^{5/2}))</th>
<th>(x_0)</th>
<th>(W_0) (fm(^4))</th>
<th>Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{16}_6)O</td>
<td>600</td>
<td>-12.72</td>
<td>2.64</td>
<td>-1.12</td>
<td>74.25</td>
<td>0.17</td>
<td>0.6</td>
<td>33%</td>
</tr>
<tr>
<td>(^{40}_20)Ca</td>
<td>650</td>
<td>-12.48</td>
<td>2.21</td>
<td>-0.77</td>
<td>71.73</td>
<td>0.13</td>
<td>0.56</td>
<td>18%</td>
</tr>
<tr>
<td>(^{48}_20)Ca</td>
<td>700</td>
<td>-12.31</td>
<td>1.88</td>
<td>-0.49</td>
<td>69.8</td>
<td>0.1</td>
<td>0.53</td>
<td>18%</td>
</tr>
<tr>
<td>(^{208}_82)Pb</td>
<td>750</td>
<td>-12.18</td>
<td>1.62</td>
<td>-0.28</td>
<td>68.28</td>
<td>0.08</td>
<td>0.51</td>
<td>38%</td>
</tr>
<tr>
<td>SkM(^*)</td>
<td>-13.4</td>
<td>2.08</td>
<td>-0.68</td>
<td>79</td>
<td>0.09</td>
<td>0.66</td>
<td>0%</td>
<td></td>
</tr>
</tbody>
</table>

### Spin-orbit spacings

<table>
<thead>
<tr>
<th>Neutrons (exp)</th>
<th>Neutrons (QMC)</th>
<th>Protons (exp)</th>
<th>Protons (QMC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{16}<em>6)O, (1p</em>{1/2} - 1p_{3/2})</td>
<td>6.10</td>
<td>6.01</td>
<td>6.3</td>
</tr>
<tr>
<td>(^{40}<em>20)Ca, (1d</em>{5/2} - 1d_{5/2})</td>
<td>6.15</td>
<td>6.41</td>
<td>6.00</td>
</tr>
<tr>
<td>(^{48}<em>20)Ca, (1d</em>{1/2} - 1d_{5/2})</td>
<td>6.05 (Sly4)</td>
<td>5.64</td>
<td>6.06 (Sly4)</td>
</tr>
<tr>
<td>(^{208}<em>82)Pb, (2d</em>{3/2} - 2d_{5/2})</td>
<td>2.15 (Sly4)</td>
<td>2.04</td>
<td>1.87 (Sly4)</td>
</tr>
</tbody>
</table>

• Neutron stars \((\text{J. Rikovska Stone et al})\)

- Hard EOS
- Maximum mass \(1.9 - 2.1 \, M_\odot\)
- \(\rho_c < 6 \, \rho_0\)
- No hyperons below \(3 \, \rho_0\)
4- Chiral effective theory and confinement

G.C. M. Ericson

a- The problem of matter stability in chiral theories

"Shifted" vacuum with chiral order parameter

\[ \bar{S} < f_\pi \]

Energy density:

\[ \epsilon(\rho, \bar{S}) = \sum_{p<p_F} \sqrt{p^2 + M_N^*(\bar{S})} + V(\bar{S}) + C_V \rho^2 \]

- NJL (Bentz, Thomas, Birse): Nucleon \( \equiv \) \( qq \) \(-\) \( q \) state. \( M_q^* = g_q S \)

- Chiral QHD: \( S = f_\pi + s \equiv \) chiral invariant scalar field: \( M_N^* = g_{sNN} \bar{S} \)

\[ g_{sNN}^*(\bar{S}) = \frac{\partial M_N^*}{\partial \bar{S}} \]

Needed to stabilize nuclear matter

- NJL: Infrared cutoff: simulate confinement
- QMC: Polarization of the quark WF: nucleon structure

Nuclear saturation vs Nucleon structure and QCD (lattice)
**b- Mean-field**

**Nucleon in the s field, radial fluctuation of the condensate**

In medium nucleon mass and energy

\[
M_N^*(\tilde{s}) = M_N \left(1 + \frac{\tilde{s}}{f_\pi}\right) + \frac{1}{2}\kappa_{NS} \tilde{s}^2
\]

\[
E_p^*(\tilde{s}) = \sqrt{\tilde{p}^2 + M_N^2(\tilde{s})}
\]

**Energy density**

\[
\frac{E_0}{V} = \varepsilon_0 = \int \frac{4d^3p}{(2\pi)^3} \Theta(p_F - p) E_p^*(\tilde{s}) + V(\tilde{s}) + \frac{g_\omega^2}{2m_\omega^2}\rho^2
\]

The nuclear scalar field \(s = S - f_\pi\) represents the dropping the chiral order parameter around the minimum of the Mexican hat effective potential

\[
V(s) = \frac{1}{2}m_\sigma^2 \left(s^2 + \frac{1}{2}\frac{s^3}{f_\pi} + \ldots\right)
\]

**with**

\[
s = -\frac{g_S^*}{m_\sigma^2}\rho_S < 0
\]

\[
m_{\sigma}^* = \frac{\partial^2 \varepsilon}{\partial s^2} = m_\sigma^2 \left(1 + \frac{3s}{f_\pi} + \frac{3}{2} \left(\frac{s}{f_\pi}\right)^2\right) + \kappa_{NS} \rho_S
\]

**ATTRACTIVE TADPOLE**: destroys saturation + chiral mass dropping  
**SCALAR RESPONSE OF THE NUCLEON**: three body repulsive force restores matter stability and stabilizes the sigma and nucleon masses
Pi-loop corrections to the Chiral condensate and Scalar susceptibility

**On top of mean field:**

\[ E_{\text{loop}} = E - E_0 \equiv V_{\varepsilon_{\text{loop}}} = \frac{3}{2} V \int_{-\infty}^{\infty} \int \frac{d\lambda}{(2\pi)^3} \int_0^1 \frac{d\lambda}{\lambda} \left( [V_L(\omega, q) \Pi_L(\omega, q; \lambda)] + 2 [V_T(\omega, q) \Pi_T(\omega, q; \lambda)] \right) \]

- \( V_L = \text{Pion + short range (} g' \text{)} \)
- \( V_T = \text{Rho + short range (} g' \text{)} \)

**Full (RPA) spin-isospin polarization propagators:**

\[ \Pi_{L,T}: \text{full (RPA) spin-isospin polarization propagators} \]

**Chiral condensate**

\[ \langle \bar{q}q \rangle = \frac{1}{2} \left( \frac{\partial\omega}{\partial m} \right)_\mu = \langle \bar{q}q \rangle_{\text{vac}} \left( 1 + \frac{s}{f_\pi} - \frac{\langle \Phi^2 \rangle}{2 f_\pi^2} \right) \]

**Scalar susceptibility**

\[ \chi_S = \left( \frac{\partial \langle \bar{q}q \rangle}{\partial m} \right)_\mu = \chi_{S,\text{nuclear}}^{\text{pionloop}} \]

\[ \chi_{S,\text{nuclear}} = -2 \frac{\langle \bar{q}q \rangle^2_{\text{vac}}}{f_\pi^2} \left( \frac{1}{m_{\sigma}^2} - \frac{1}{m_{\sigma^{'}}^2} \left( \frac{\sigma_{N}^{(\pi)} + \sigma_{N}^{(\sigma)}}{\sigma_{N}^{(\sigma)}} \right)^2_{\text{eff}} \right) \Pi_{SS}(0) \frac{1}{m_{\sigma^{'}}^2} \]

\[ \chi_{S,\text{pionloop}} = -2 \frac{\langle \bar{q}q \rangle^2_{\text{vac}}}{f_\pi^4} \int \frac{id^4q}{(2\pi)^4} \left( D_\pi D_{0\pi}^2 + D_{\pi}^2 D_{0\pi} \right)(q) \tilde{n}^0(q) \]

\[ \tilde{n}^0(q) = \frac{n^0(q)}{1 - g' n^0(q)} \]
d- Fixing the parameters; analysis of lattice data

Lattice data: low quark (pion) mass not available + difficulties with non analytic behaviour of pion loops

\[ M_N(m_\pi^2) = a_0 + a_2 m_\pi^2 + a_4 m_\pi^4 + \Sigma_\pi(m_\pi, \Lambda) \]

Extrapolate lattice data with well controlled chiral model depending on a cutoff (nucleon size)

\[ \Lambda = 980 \text{ MeV} \]
\[ \Sigma_\pi = -421.6 \text{ MeV} \]
\[ \sigma_\pi^{(\pi)} = 21.5 \text{ MeV} \]

Best fit

\[ a_2 \approx 1.5 \text{ GeV}^{-1} \quad a_4 \approx -0.5 \text{ GeV}^{-3} \]

(Leinweber et al)

\[ \sigma_{\text{non-pion}}^{\text{non-pion}} = m_\pi^2 \frac{\partial M}{\partial m_\pi^2} = a_2 m_\pi^2 + 2 a_4 m_\pi^4 \approx 29 \text{ MeV} = f_\pi g_s \frac{m_\pi^2}{m_\sigma^2} \]

\[ m_\sigma = 800 \text{ MeV} \]

\[ \chi_{\text{NS}}^{(\sigma)} = -2 \frac{\langle \bar{q}q \rangle_{\text{vac}}^2}{f_\pi^2} \frac{1}{m_\sigma^2 - m_\rho^2} \left( \frac{m_\rho^2}{m_\sigma^2} - \frac{1}{4} \right) \]

\[ a_4 = \frac{f_\pi g_s}{2 m_\sigma^4} (3 - 2 C) \]

\[ C' = \left( \frac{f_\pi^2}{2 M_N} \right) \kappa_{\text{NS}} \approx 1.25 \]
The sigma mass remains stable

Higher densities?
Phase transition to quark matter?

\[ m_\sigma = 850 \text{ MeV} \quad g_\omega = 8 \]
\[ C = 0.985 + \text{Density dependence} \]
5- Towards High temperature or baryonic densities

**ISSUES**
- Chiral symmetry restoration and deconfinement
- (Tri)critical point?
- Hadrons near phase transition?

**SIGNATURES**
- Bulk thermodynamic variables
- In-medium hadron spectral functions
- Charm, dileptons

**THEORETICAL TOOLS**
- Lattice QCD at finite $\mu$
- Effective theories
- Renormalization group

**Diagram**
- T vs. $\mu$
- Phase transition regions
- Quark-gluon plasma
- Hadron gas
- Color superconductor
- In medium effects

**Institutional Logos**
- RHIC
- LHC/ALICE
- FAIR/CBM
To study chiral transition to quark matter: chiral Theory incorporating confinement at the quark level

• **Diquarks:** (qq in $\frac{3}{c}$) Nucleon=quark-diquark bound state
  Diquark condensate: CSC phase

• **Attempt (Lawley, Bentz, Thomas):** NJL model including diquark interaction and (kind of confinement)

  1- **Low $T$, $\rho$:** Spontaneous chiral symmetry breaking: quark condensate

  \[ M = m - 2G_1 \langle \bar{q}q \rangle \]

  2- **Nucleon:** quark + diquark bound state + Confinement (IR cut-off generates a scalar susceptibility) + pion cloud

\[ \tau_s(q) = \frac{4iG_s}{1 + 2G_s \Pi_s(q)} \]

\[ T(p) = \frac{3}{M} \frac{1}{1 + \frac{3}{M} \Pi_N(p)} \]

\[ M_B = M_B^{(0)}(q + \text{diquarks}) + \Sigma_B(\pi \text{ cloud}) \]
3- Phase diagram (matter in $\beta$ equilibrium)

**Nuclear matter**

V\text{NM} = V_{\text{vac}} + V_N - \frac{\omega^2_0}{4G_\omega} - \frac{\rho^2_0}{4G_\rho} - \frac{\mu_e^4}{12\pi^2}

\mu_B = \frac{1}{2}(\mu_p + \mu_n); \quad \mu_I = \frac{1}{2}(\mu_p - \mu_n); \quad \mu_e = \mu_n - \mu_p

**Quark matter**

V\text{QM} = V_{\text{vac}} + V_Q + V_\Delta - \frac{\mu_e^4}{12\pi^2}

\mu_B = \frac{3}{2}(\mu_u + \mu_d); \quad \mu_I = \frac{1}{2}(\mu_u - \mu_d); \quad \mu_e = \mu_d - \mu_u

**Stable nuclear matter**

$M_0=400$ MeV, IR cutoff

Nucleon $r_s = G_S/G_1$ (diquark) vs $\Sigma_N$ (pion cloud)

**Nuclear matter:** $G_\omega$, $G_\rho$ (saturation, asymmetry energy)

$\Sigma_N=0$: only diquark

To bind the nucleon

$\Sigma_N=-300$ MeV

**Neutron star**

No nuclear matter!
**b-Chiral symmetry restoration vs deconfinement**

- **Polyakov loops**

\[
\Phi = \frac{1}{3} \text{Tr}_c L
\]

\[
L(\vec{x}) = \mathcal{P} \exp \left[ i \int_0^\beta d\tau A_4(\vec{x}, \tau) \right]
\]

Order parameter associated with confinement for pure gauge QCD

- **Low T (confined) phase:** \( \langle \Phi \rangle = 0 \)
- **High T (deconfined) phase:** \( \langle \Phi \rangle \to 1 \)

- **PNJL model** *(Fukushima, Ratti et al, Hansen et al, Sazaki et al)*

\[
\mathcal{L}_{PNJL} = \bar{q} \left( i \gamma_\mu D^\mu - \hat{m}_0 \right) q + G_1 \left[ (\bar{q}q)^2 + (\bar{q}i\gamma_5 \tau q)^2 \right] - \mathcal{U} (\Phi[A], \bar{\Phi}[A]; T)
\]

Quarks coupled to background gluon field representing Polyakov loops dynamics

\[
D^\mu = \partial^\mu - iA^\mu
\]

\[
A^\mu = \delta^\mu_0 A^0
\]

\[
A^0 = -iA^4
\]
• Quark number density (and susceptibilities)

\[ f_\Phi^+(E_p) = \frac{\langle \Phi \rangle + 2\Phi e^{-\beta(E_p-\mu)}}{1 + 3\langle \Phi \rangle e^{-\beta(E_p-\mu)}} \]

High T
\[ \langle \Phi \rangle \rightarrow 1 \]

Low T
\[ \langle \Phi \rangle = 0 \]

\[ f_\Phi^+(E_p) = \frac{1}{(e^{(E_p-\mu)/T} + 1)} \]

PNJL mimics three-quarks clustering

• Sigma and pion spectral functions

(Hansen et al)

\[ T < T_c, \sigma \text{ width decreases} \]

\[ T > T_c, \sigma \text{ width increases} \]

\[ \mu = 0.6 T_c \]

\[ T_{dissoc} \text{ decreases} \]
IV- Dileptons production in relativistic heavy ion collisions
1. Theoretical approaches

• Density expansion
• Many-body approaches
• Transport codes
• QCD sum rules
• Weinberg sum rules
• ........
• Renormalization group

Current current correlator in the vector channel

\[ \Pi_{\mu \nu}^{\mu \nu}(q) = -i \int d^4 x e^{-i q x} \langle \langle \mathcal{V}^\mu(x), \mathcal{V}^\nu(0) \rangle \rangle(T, \rho_B) \]

\[ \frac{dN_{ll}}{d^4 x d^4 q} = -\frac{\alpha^2}{6\pi^3 M^2} \frac{1}{e^{\beta q^0} - 1} g_{\mu\nu} \left( \frac{1}{\pi} \text{Im} \Pi_{\mu\nu}^{\mu\nu} \right) \]
a- Introduction: basic features

• Low mass dileptons at CERN/SPS
  \(0.3 \text{ GeV} < M < 0.6 \text{ GeV}\)
  Important radiation beyond the conventional sources: the hadronic « cocktail »

\[ \rho, \omega, \Phi \rightarrow \ell \bar{\ell} \]

\[ \eta, \eta' \rightarrow e^+ e^- \gamma, \quad \omega \rightarrow e^- e^+ \pi_0 \]

Free annihilation on top of final state decay not sufficient

Strong in-medium effects in the fireball

• Intermediate mass
  \(1 \text{ GeV} < M < 2 \text{ GeV}\)
  Large excess vs (p, A)

Formation of dense and hot matter?
- QGP and/or quark–hadron duality
- Hot hadronic matter
\[ \pi a_1 \rightarrow \ell \bar{\ell} \]
**b- Dilepton production rate**

\[
\frac{dR}{d^4x \, d^4q} = -\frac{\alpha^2}{6\pi^3 M^2} g_{\mu\nu} W^{\mu\nu}(q)
\]

**Physics embedded in the hadronic tensor**

\[
W^{\mu\nu}(q) = \sum_i e^{-\beta E_i} \frac{1}{Z} \sum_f <i|J^{\nu}(0)|f> <f|J^{\mu}(0)|i> (2\pi)^4 \delta(q_0 + p_f - p_i) \delta^{(3)}(q + p_f - p_i)
\]

Can be expressed in terms of other tensors with non trivial T=0 limit

\[
\frac{W^{\mu\nu}(q)}{2\pi} = e^{-\beta q_0} \rho^{\mu\nu}(q) = \frac{(-\frac{1}{4}) \text{Im} \Pi^{\mu\nu}_R(q)}{e^{\beta q_0} - 1} = \frac{(-\frac{1}{4}) \text{Im} \Pi^{\mu\nu}(q)}{e^{\beta q_0} + 1}
\]

\[
\rho^{\mu\nu}(q) = \sum_f e^{-\beta E_f} \frac{1}{Z} \sum_i <f|J^{\nu}(0)|i> <i|J^{\mu}(0)|f> (2\pi)^4 \delta(q_0 + p_f - p_i) \delta^{(3)}(q + p_f - p_i)
\]

**Emission**  \[ i \rightarrow f + \bar{l}l \]  \[ \bar{l}l \rightarrow i \]  **Annihilation**  \[ f + \bar{l}l \rightarrow i \]**Photon absorption**

**Current-current correlators**

\[
\Pi^{\mu\nu}_R(q) = -i \int d^4 x e^{i q \cdot x} \Theta(x_0) \langle \langle [J^{\mu}(x), J^{\nu}(0)] \rangle \rangle
\]

\[
\Pi^{\mu\nu}(q) = -i \int d^4 x e^{i q \cdot x} \langle \langle \mathcal{T} (J^{\mu}(x), J^{\nu}(0)) \rangle \rangle
\]

**Field theory at finite T**

**Many-body approach**

**Density expansion**
**c- Vacuum correlator**

\[ \Pi_{\text{vide}}^{\mu \nu}(q) = - \left( g^{\mu \nu} - \frac{q^\mu q^\nu}{q^2} \right) \Pi(q^2) \]

Known from e+e- annihilation

\[ \frac{dR}{d^4 x d^4 q} = - \frac{\alpha^2}{\pi^3 q^2} \frac{1}{e^{\beta q_0} + 1} \text{Im} \Pi(q^2) \]

**Quark basis vs hadron basis**

\[ R(s) = \frac{\sigma(e^+ e^- \rightarrow \text{hadrons})}{\sigma(e^+ e^- \rightarrow \mu^+ \mu^-)} = - \frac{12 \pi}{s} \text{Im} \Pi(s) \]

**Spectral function Im \( \Pi(s) \)**

\[ J^\mu = \frac{2}{3} \bar{u} \gamma^\mu u - \frac{1}{3} \bar{d} \gamma^\mu d - \frac{1}{3} \bar{s} \gamma^\mu s = J^\mu_\rho + J^\mu_\omega + J^\mu_\Phi \]

\[ J^\mu_\rho = \frac{1}{2} (\bar{u} \gamma^\mu u - \bar{d} \gamma^\mu d), \quad J^\mu_\omega = \frac{1}{6} (\bar{u} \gamma^\mu u + \bar{d} \gamma^\mu d), \quad J^\mu_\Phi = -\frac{1}{3} \bar{s} \gamma^\mu s \]

\[ \sum_{\rho, \omega, \phi} \left[ \frac{m_V^2}{g_V} \right]^2 \text{Im} D_V(s) \]

\[ \sum \left( e_q \right) \left[ 1 + \frac{\alpha_s(s)}{\pi} + \ldots \right] \]

\[ \frac{-s}{12\pi} N_c \sum_{u,d,s} (e_q)^2 \left[ 1 + \frac{\alpha_s(s)}{\pi} + \ldots \right] \]

**s < s_{\text{Dual}} \sim (1.5\text{GeV})^2:**

\( e^- e^+ \rightarrow \rho \rightarrow \pi \pi \)
\( e^- e^+ \rightarrow \omega \rightarrow 3 \pi \)
\( e^- e^+ \rightarrow \Phi \rightarrow K \bar{K} \)

\( \rho:9 \)
\( \omega:1 \)
\( \Phi:2 \)

\( \sum \left( e_q \right) \left[ 1 + \frac{\alpha_s(s)}{\pi} + \ldots \right] \]

\[ \frac{-s}{12\pi} N_c \sum_{u,d,s} (e_q)^2 \left[ 1 + \frac{\alpha_s(s)}{\pi} + \ldots \right] \]

**s > s_{\text{Dual}}:**

\[ \frac{-s}{12\pi} N_c \sum_{u,d,s} (e_q)^2 \left[ 1 + \frac{\alpha_s(s)}{\pi} + \ldots \right] \]
d- Density expansion

\[ \Pi^{\mu\nu}(q, T, \mu) = \Pi^{\mu\nu}_{\text{vac}}(q) + \sum_h \rho_h \Pi^{\mu\nu}_h(q) \]

\[ \int \frac{d^3k}{(2\pi)^3} \frac{f(\omega_k)}{2\omega_k} \int d^4x e^{iqx} \left< \pi(k) | T(J^\mu(x), J^\mu(0)) | \pi(k) \right> \]

\[ \int \frac{d^3p}{(2\pi)^3} \frac{f(E_p) M}{E_p} \int d^4x e^{iqx} \left< N(p) | T(J^\mu(x), J^\mu(0)) | N(p) \right> \]

- Chiral reduction formalism (chiral symmetry + empirical data) \cite{Zahed et al}

\[
\frac{dR}{d^4xd^4q} = -\frac{\alpha^2}{\pi^3 q^2} \frac{1}{e^{3q_0} + 1} \left[ Im \Pi(q^2) - \frac{2}{f^2} \int \frac{d\omega_k}{(2\pi)^3} \frac{n(\omega_k)}{\omega_k} Im \Pi_V(q^2) \right.
\]

\[
+ \frac{1}{f^2} \int \frac{d\omega_k}{(2\pi)^3} \frac{n(\omega_k)}{\omega_k} \left( Im \Pi_A((q+k)^2) + Im \Pi_A((q-k)^2) \right) + \ldots \]

Axial-vector mixing: \( \gamma^*\pi \rightarrow a_1 \), \( \gamma^*\pi \rightarrow \text{Axial nucleonic current} \)

Increase the DPR below the rho bump

Soft pion limit \((k \ll \omega)\)

\[ \Pi^{\mu\nu}_V(q; T) = (1 - \epsilon) \Pi^{\mu\nu}_V(q; T = 0) + \epsilon \Pi^{\mu\nu}_A(q; T = 0) \]

\[ \epsilon = \frac{T^2}{6 \frac{f^2}{f^2}} = \frac{2}{f^2} \int \frac{d\omega_k}{(2\pi)^3} \frac{n(\omega_k)}{\omega_k} = \frac{2}{3} \frac{\langle \Phi^2 \rangle}{f^2} \]

| Pions | \( \gamma^*\pi \rightarrow \gamma^*\pi \) |
| Nucleons | \( \gamma^*N \) absorption |

\(--\--\)
**e. Many body approach** (R. Rapp, J. Wambach, G. C)

**ρ propagator**

\[ D_\rho(M, q, \mu_{B,T}) = \left( M^2 - m_\rho^2 - \Sigma_{\rho\pi\pi} - \Sigma_{\rho_B} - \Sigma_{\rho_M} \right)^{-1} \]

**ρ self-energy**

\[ \Sigma_{\rho\pi\pi} = \begin{array}{c} \rho \\ \Sigma_\pi \\ \rho \\ \Sigma_\pi \\ \rho \end{array} \]

\[ \Sigma_{\rho_B,M} = \begin{array}{c} \rho \\ B^*, a_1, K_1 \cdots \\ \rho \\ N, \pi, K \cdots \end{array} \]

**Constraints:**

- vacuum decays: \( B,M \rightarrow \rho N, \rho \pi, \cdots \)
- scattering data: \( \gamma N, \gamma A, \pi N \rightarrow \rho N \)
- QCD sum rules

\[ \sigma_{\gamma A}^{abs}(q_0)/A \propto \text{Im}D_\rho(q_0=q) \]

*Urban et al*
In-medium $\pi$ cloud

- P. Schuck, G.C, 92

In-medium $\pi$ cloud + $\rho N \rightarrow B^*$

- Urban et al, 98
- $q=0$

$\rho N \rightarrow B^*$ + meson gas

- Post et al, 02
- $q=0 \ GeV$

Strong broadening/melting of the $\rho \rightarrow$ pQCD continuum
Baryon density more important than $T$
Baryon effects important even at $\rho_{B,\text{tot}} = 0$:
- sensitive to $\rho_{B,\text{tot}} = \rho_B + \rho_{\bar{B}}$ ($\rho$-$N$ and $\rho$-$\bar{N}$ interactions identical)
- $\omega$ also melts, $\Phi$ more robust
From SPS to LHC (R. Rapp)

From SPS to LRHIC

Dilepton Excitation Function in Central Au-Au ($N_{\text{part}} = 330$)

- $|y_e| < 0.5$, $p_t > 0.2 \text{GeV}$
- Hadronic Matter
- QGP

RHIC

Central Au+Au $s^{1/2} = 200 \text{AGeV}$

- $\langle N_{\text{ch}} \rangle = 800$
- Drell-Yan
- Open Charm
- QGP
- Thermal

Central Pb+Pb $s^{1/2} = 5.5 \text{ATeV}$

- $\langle N_{\text{ch}} \rangle = 3000$
- Charm (est.)
- QGP (ch-eq)
- Thermal (tot)
**f- Dropping ρ-meson mass and scaling laws**

- **a- Scale invariance of QCD** *(Brown-Rho 91)*
  
  Introduce Dilaton field
  
  $\chi_0^* = \left( \frac{<G \cdot G^*>}{<G \cdot G>} \right)^{1/4} = \left( \frac{<\bar{q}q^*>}{<\bar{q}q>} \right)^{1/3} = \frac{f_\pi^*}{f_\pi} = \frac{m_V^*}{m_V}$

  **Low density**
  
  $= 1 - 0.12 \frac{\rho}{\rho_0}$

- **b- Vector manifestation of chiral symmetry** *(Harada-Yamawaki)*
  
  - Based on a very promising renormalization group approach + matching of the EFT to QCD at $\Lambda_0 \approx 1 \text{ GeV}$
  
  - Worked out explicitly in a model (HLS) where the ρ meson is introduced as the gauge boson of a hidden local symmetry
  
  - Matching of the axial and vector correlators at $\Lambda_0 \approx 1 \text{ GeV}$
    
    $\Pi_{V,A}^{HLS}(\Lambda_0) = \Pi_{V,A}^{QCD}(\Lambda_0)$
  
  - Renormalization group equation:
    
    $f_\pi(\Lambda), \quad a(\Lambda), \quad g(\Lambda)$

VDM recovered numerically in vacuum

But fate of VDM at finite T and $\mu$ !
**g- Lowering of the quark-hadron duality threshold as a signature of chiral restoration**

- Hard-Thermal-Loop result much enhanced over Born rate
- "matching" of HG and QGP automatic!
- Quark-Hadron Duality at low mass ?!
- Degenerate axial and vector correlators?

[Braaten, Pisarski+Yuan '90]

\[ [qq \to ee] \]
\[ [qq+\alpha(\alpha_s)-HTL] \]
2- Comparison with data

α- Pb–Au collisions at CERN/SPS: CERES/NA45

Top SPS Energy

\[
\begin{aligned}
(T_{in} = 190 \text{ MeV}) & \rightarrow (T_{exp} = 170 \text{ MeV}) & \rightarrow (T_{f,a} = 115 \text{ MeV}) \\
(\rho_B)_{in} = 2.55 \rho_0 & \rightarrow (\rho_B)_{exp} = 270 \rho_0 & \rightarrow (\rho_B)_{f,a} = 0.33 \rho_0
\end{aligned}
\]

QGP contribution small

Medium effects on $\rho$ meson

Evolve dilepton rates over thermal fireball QGP+Mix+HG (Rapp et al):

• Rho meson melts in dense matter
• Baryon density more important than temperature (40A. GeV vs 158A. GeV)
• Hades data/ Futur GSI: CBM (~ 30A.GeV)
NA60 has *extracted* the rho meson spectral function.

**Free spectral function ruled out**

**Meson gas insufficient**

Consistent with the modification (broadening) of the rho meson spectral function

(Rapp-Wambach/Chanfray)

Simplistic dropping mass ruled out

\[
\frac{m_\rho(T,\rho_B)}{m_\rho^{\text{vac}}} = \left[1 - \left(\frac{T}{T_c}\right)^2\right]^{\alpha} \left[1 - C \frac{\rho_B}{\rho_0}\right]
\]

The dropping mass underestimates the peak region

RG approach (HLS) might modify this conclusion! *(Brown, Rho)*
**Dilepton spectra**

\[
\frac{dN_{ee}^{\text{therm}}}{dM} = \int_{\tau_0}^{\tau_f} d\tau V_{FB}(\tau) \int \frac{Md^3q}{q_0} dN_{ee}^{\text{therm}}(M,q;T,\mu_i) \left[ \exp(\mu_\pi/T) \right]^N \pi \text{Acc}
\]

- Expanding fireball with entropy and baryon number conservation which fixes \(T(\mu_B)\) in the phase diagram:
  \[T_0=197\text{ MeV (QGP)} \rightarrow (\mu_B^0, T_c)=(232, 175)\text{ MeV} \rightarrow \text{Hadronic phase with } \mu_\pi \text{ } \rightarrow \]
  \(T_0 \approx 120\text{ MeV}\)

- Medium effects on \(\omega\) and \(\Phi + 4\pi\) and axial-vector mixing

**Many-body+fireball**

**Chiral virial expansion + hydro**

---

**Rapp, van Hees**

**Dusling et al**

**Lack of broadening!**
c- HADES data

G. Agakichiev et al, subm. to PRL

\[ ^{12}\text{C} + ^{12}\text{C} \rightarrow 2A \text{GeV} \]
\[ \theta_{ee} > 9^\circ \]

**cocktail A**: post freeze-out hadronic sources:
\[ \pi^0, \eta \rightarrow \gamma \, e^+e^-; \, \omega \rightarrow \pi^0 \, e^+e^-; \, \omega \rightarrow e^+e^-; \]

**cocktail B**: additional fireball sources:
\[ \Delta \rightarrow N \, e^+e^-; \, \rho \rightarrow e^+e^-; \]

**sizable excess in dilepton yield relative to contributions from long-living mesons**

**origin of excess-yield??**
Comparison with transport codes
Summary

In-medium effects generated by chiral dynamics
- pions, sigma, omega, kaons, rho

Mechanisms and manifestations of chiral restoration
- Dropping mass and radial fluctuations of the condensate
- Axial vector mixing driven by pions (the pion scalar density)
- QCD (chiral) susceptibilities at finite T and ρ

The role of hadron substructure and confinement
- Dropping of the condensate and hadron substructure
- Scalar response of the nucleon and the EOS
- The diquarks from nucleon to color superconducting phase, Polyakov loops

Dileptons production
- Radiation from a chiral restored phase?
- but the compatibility with hadronic model without coupling to condensate is unexpected!
Additionnal slides
DFT approach: Mean-field + ChiPT + Pairing (Gogny)

Shape coexistence is predicted in agreement with Skyrme and Gogny calculation

*Finelli et al*

Fig. 12. Binding energy curves of even-\(A\) Pb isotopes as functions of the quadrupole deformation \(\beta_2\). The curves correspond to RHB calculations with constrained quadrupole deformation.
Complement on PNJL model \((\text{Roessner et al})\)

\[ c_2 = \frac{1}{2} \frac{\partial^2 (p/T^4)}{\partial (\mu/T)^2} \bigg|_{\mu=0}, \quad c_4 = \frac{1}{24} \frac{\partial^4 (p/T^4)}{\partial (\mu/T)^4} \bigg|_{\mu=0} \]
4.2 Many-Body Approach: \( \rho \)-Meson in Vacuum

Introduce \( \rho \) as gauge boson into free \( \pi + \rho \) Lagrangian \( \Rightarrow \)

\[
\mathcal{L}^\text{int}_{\pi\rho} = g \bar{\rho}_\mu (\bar{\pi} \times \partial^\mu \pi) - \frac{1}{2} g^2 \bar{\rho}_\mu \bar{\rho}^\mu \bar{\pi} \cdot \bar{\pi}
\]

\[
\Sigma_\rho^{\mu\nu}(q^2) = g^2 \int \frac{d^4k}{(2\pi)^4} D_\pi(k) \nu^\mu D_\pi(k+q) \nu^\nu + 2g^2 g^{\mu\nu} \int \frac{d^4k}{(2\pi)^4} D_\pi(k) \quad q_\mu \Sigma_\rho^{\mu\nu} = 0
\]

\( \rho \)-propagator:

\[
D_\rho(M) = \left[ M^2 - (m_\rho^{(0)})^2 - \Sigma_\rho\pi\pi(M) \right]^{-1}
\]

\( \pi \) e.m. formfactor

\[
|F_\pi(M)|^2 = (m_\rho^{(0)})^4 |D_\rho(M)|^2
\]

\( \pi\pi \) scattering phase shift

\[
\delta_{\pi\pi}(M) = \tan^{-1} \left( \frac{\text{Im}D_\rho(M)}{\text{Re}D_\rho(M)} \right)
\]
4.2.2 $\rho$-Selfenergies in Hot + Dense Matter

modifications due to interactions with hadrons from heat bath
$\Rightarrow$ In-Medium $\rho$-Propagator

$$D_{\rho}(M,q;\mu_B,T)=[M^2-m_{\rho}^2-\Sigma_{\rho\pi\pi}-\Sigma_{\rho B}-\Sigma_{\rho M}]^{-1}$$

(1) Medium Modifications of Pion Cloud

$$\Sigma_{\rho\pi\pi} = \int D_{\pi}^{med} \nu_{\rho\pi\pi} D_{\pi}^{med} (1+2f_{\pi}) + \int D_{\pi}^{med} \nu_{\rho\pi\pi}(1+f_{\pi})$$

In-med $\pi$-prop.

$$D_\pi = [k_0^2 - \omega_k^2 - \Sigma_{\pi}(k_0,k)]^{-1}$$

$\Rightarrow$ mostly affected by (anti-) baryons

[Chanfray et al, Herrmann et al, RR et al, Weise et al, Oset et al, …]
**Direct $\rho$-Hadron Interactions**

Resonance-dominated: $\rho + h \rightarrow R$, self-energy:

$$\Sigma_{\rho hR} = \int \frac{d^3k}{(2\pi)^3} D_R(k+q) v^2_{\rho hR} \left[ f^h(\omega_k) \pm f^R(\omega_R) \right]$$

(i) *Meson Gas* ($h = \pi, K, \rho$)

E.g. $\mathcal{L}_{\pi\rho} = GA^\mu \left( \partial^\nu \pi \right) \rho_{\mu\nu}$, $A=a_1, h_1$

Fix $G$ via $\Gamma(a_1 \rightarrow \rho\pi) \sim G^2 v^2 PS \approx 0.4\text{GeV}$.

**Generic features:**

- cancellations in real parts
- imaginary parts strictly add up

![Graph showing the behavior of various mesons](image)
(ii) **Direct ρ-Baryon Interactions (h = N, Δ, …)**

* S-wave $\rho + N \rightarrow B^{3/2-}$:

\[ \mathcal{L}_{\rho BN}^{S} = \frac{f_{\rho BN}}{m_{\rho}} \Psi_{B}^{+} (q_{0} \bar{S} \cdot \bar{\rho}_{a} - \rho_{a}^{0} \bar{S} \cdot \bar{q}) t_{a} \Psi_{N} \]

* P-wave $\rho + N \rightarrow B^{3/2+}$:

\[ \mathcal{L}_{\rho BN}^{P} = \frac{f_{\rho BN}}{m_{\rho}} \Psi_{B}^{+} (\bar{S} \times \bar{q}) \cdot \bar{\rho}_{a} t_{a} \Psi_{N} \]

**In-Medium Selfenergy:**

\[ \Sigma_{\rho BN} = - \left( \frac{f_{\rho BN} F(q)}{m_{\rho}} \right)^{2} SI Q^{2} \phi(q_{0}, q, \mu_{B}, T) \]

**Coupling Constant → Free Decay:**

\[ \Gamma_{B \rightarrow \rho N} = C f_{\rho BN}^{2} \int M dM \mathcal{A}_{\rho}(M) q_{cm} Q^{2} F(q^{2})^{2} \]

**Examples:**

- $\Gamma(N(1520) \rightarrow N\rho) \approx 25$ MeV,
- $\Gamma(\Delta(1700) \rightarrow N\rho) \approx 130$ MeV

---

**Sub-threshold Decay Phase Space**

- $\rho(770)$
- $\Delta(1700)$
- $N(1520)$
Vector and axialvector spectral functions

$$\Pi^\mu_\nu (q) = -i \int d^4x e^{iq\cdot x} \ll T (V^\mu_k (x), V^\nu_k (0)) \gg$$
$$\Pi^\mu_A (q) = -i \int d^4x e^{iq\cdot x} \ll T (A^\mu_k (x), A^\nu_k (0)) \gg$$

$$V^\mu_k = \bar{\psi} \gamma^\mu \frac{T_k}{2} \psi, \quad A^\mu_k = \bar{\psi} \gamma^\mu \gamma_5 \frac{T_k}{2} \psi,$$

Associated with chiral partners $\rho - a_1(1260)$

Chiral restoration means: vector and axialvector correlation functions become identical

An illustration: Weinberg sum rule

$$f^2_\pi = - \int \frac{ds}{\pi s} (\text{Im} \, \Pi_V - \text{Im} \, \Pi_A) \quad \rightarrow 0 \text{ at chiral restoration}$$
Old and new CERES data

- QGP contribution small
- medium effects on \( \rho \)-meson!